Cover photo credits: Left © Richard Hutchings/CORBIS; Right © Royalty-Free/CORBIS; Bottom © LWA-Dann Tardif/CORBIS.
Dear Student and Parent:

The Texas Assessment of Knowledge and Skills (TAKS) is a comprehensive testing program for public school students in grades 3–11. TAKS, including TAKS (Accommodated) and Linguistically Accommodated Testing (LAT), has replaced the Texas Assessment of Academic Skills (TAAS) and is designed to measure to what extent a student has learned, understood, and is able to apply the important concepts and skills expected at each tested grade level. In addition, the test can provide valuable feedback to students, parents, and schools about student progress from grade to grade.

Students are tested in mathematics in grades 3–11; reading in grades 3–9; writing in grades 4 and 7; English language arts in grades 10 and 11; science in grades 5, 8, 10, and 11; and social studies in grades 8, 10, and 11. Every TAKS test is directly linked to the Texas Essential Knowledge and Skills (TEKS) curriculum. The TEKS is the state-mandated curriculum for Texas public school students. Essential knowledge and skills taught at each grade build upon the material learned in previous grades. By developing the academic skills specified in the TEKS, students can build a strong foundation for future success.

The Texas Education Agency has developed this study guide to help students strengthen the TEKS-based skills that are taught in class and tested on TAKS. The guide is designed for students to use on their own or for students and families to work through together. Concepts are presented in a variety of ways that will help students review the information and skills they need to be successful on TAKS. Every guide includes explanations, practice questions, detailed answer keys, and student activities. At the end of this study guide is an evaluation form for you to complete and mail back when you have finished the guide. Your comments will help us improve future versions of this guide.

There are a number of resources available for students and families who would like more information about the TAKS testing program. Information booklets are available for every TAKS subject and grade. Brochures are also available that explain the Student Success Initiative promotion requirements and the new graduation requirements for eleventh-grade students. To obtain copies of these resources or to learn more about the testing program, please contact your school or visit the Texas Education Agency website at www.tea.state.tx.us/student.assessment.

Texas is proud of the progress our students have made as they strive to reach their academic goals. We hope the study guides will help foster student learning, growth, and success in all of the TAKS subject areas.

Sincerely,

Gloria Zyskowski
Deputy Associate Commissioner for Student Assessment
Texas Education Agency
## Mathematics

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<th>Page</th>
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What Is This Book?
This is a study guide to help you strengthen the skills tested on the Grade 7 Texas Assessment of Knowledge and Skills (TAKS). TAKS is a state-developed test administered with no time limit. It is designed to provide an accurate measure of learning in Texas schools.

By acquiring all the skills taught in seventh grade, you will be better prepared to succeed on the Grade 7 TAKS and during the next school year.

What Are Objectives?
Objectives are goals for the knowledge and skills that a student should achieve. The specific goals for instruction in Texas schools were provided by the Texas Essential Knowledge and Skills (TEKS). The objectives for TAKS were developed based on the TEKS.

How Is This Book Organized?
This study guide is divided into the six objectives tested on TAKS. A statement at the beginning of each objective lists the mathematics skills you need to acquire. The study guide covers a large amount of material. You should not expect to complete it all at once. It may be best to work through one objective at a time.

Each objective is organized into review sections and a practice section. The review sections present examples and explanations of the mathematics skills for each objective. The practice sections feature mathematics problems that are similar to the ones used on the TAKS test.

How Can I Use This Book?
First look at your Confidential Student Report. This is the report the school gave you that shows your TAKS scores. This report will tell you which TAKS subject-area test(s) you passed and which one(s) you did not pass. Use your report to determine which skills need improvement. Once you know which skills need to be improved, you can read through the instructions and examples that support those skills. You may also choose to work through all the sections. Pace yourself as you work through the study guide. Work in short sessions. If you become frustrated, stop and start again later.
What Are the Helpful Features of This Study Guide?

- There are several words in this study guide that are important for you to understand. These words are boldfaced in the text and are defined when they are introduced. Locate the boldfaced words and review the definitions.

- Examples are contained inside shaded boxes.

- Each objective has “Try It” problems based on the examples in the review sections.

- A Grade 7 Mathematics Chart is included on pages 8–9 and also as a tear-out page in the back of the book. This chart includes useful mathematics information. The tear-out Mathematics Chart in the back of the book also provides both a metric and a customary ruler to help solve problems requiring measurement of length.

- Look for the following features in the margin:
  Ms. Mathematics provides important instructional information for a topic.

  Detective Data offers a question that will help remind you of the appropriate approach to a problem.

  Do you see that . . . points to a significant sentence in the instruction.
How Should the “Try It” Problems Be Used?

“Try It” problems are found throughout the review sections of the mathematics study guide. These problems provide an opportunity for you to practice skills that have just been covered in the instruction. Each “Try It” problem features lines for your responses. The answers to the “Try It” problems are found immediately following each problem.

While completing a “Try It” problem, cover up the answer portion with a sheet of paper. Then check the answer.

What Kinds of Practice Questions Are in the Study Guide?

The mathematics study guide contains questions similar to those found on the Grade 7 TAKS test. There are two types of questions in the mathematics study guide.

- Multiple-Choice Questions: Most of the practice questions are multiple choice with four answer choices. These questions present a mathematics problem using numbers, symbols, words, a table, a diagram, or a combination of these. Read each problem carefully. If there is a table or diagram, study it. You should read each answer choice carefully before choosing the best answer.

- Griddable Questions: Some practice questions use a seven-column answer grid like those used on the Grade 7 TAKS test.

How Do You Use an Answer Grid?

The answer grid contains seven columns, which includes two decimal places: tenths and hundredths.

Suppose 3,108.6 is the answer to a problem. First write the number in the blank spaces. Be sure to use the correct place value. For example, 3 is in the thousands place, 1 is in the hundreds place, 0 is in the tens place, 8 is in the ones place, and 6 is in the tenths place.

Then fill in the correct bubble under each digit. Notice that if there is a zero in the answer, you need to fill in the bubble for the zero.

The grid shows 3,108.6 correctly entered. The zero in the tens place is bubbled in because it is part of the answer. It is not necessary to bubble in the zero in the hundredths place, because this zero will not affect the value of the correct answer.

Where Can Correct Answers to the Practice Questions Be Found?

The answers to the practice questions are in the answer key at the back of this book (pages 145–153). Each question includes a reference to the page number in the answer key for the answer to the problem. The answer key explains the correct answer, and it also includes some explanations for incorrect answers. After you answer the practice questions, you can check your answers.

If you still do not understand the correct answer after reading the answer explanations, ask a friend, family member, or teacher for help. Even if you have chosen the correct answer, it is a good idea to read the answer explanation because it may help you better understand why the answer is correct.
# Grade 7 Mathematics Chart

## LENGTH

<table>
<thead>
<tr>
<th>Metric</th>
<th>Customary</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 kilometer = 1000 meters</td>
<td>1 mile = 1760 yards</td>
</tr>
<tr>
<td>1 meter = 100 centimeters</td>
<td>1 mile = 5280 feet</td>
</tr>
<tr>
<td>1 centimeter = 10 millimeters</td>
<td>1 yard = 3 feet</td>
</tr>
<tr>
<td></td>
<td>1 foot = 12 inches</td>
</tr>
</tbody>
</table>

## CAPACITY AND VOLUME

<table>
<thead>
<tr>
<th>Metric</th>
<th>Customary</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 liter = 1000 milliliters</td>
<td>1 gallon = 4 quarts</td>
</tr>
<tr>
<td></td>
<td>1 gallon = 128 fluid ounces</td>
</tr>
<tr>
<td></td>
<td>1 quart = 2 pints</td>
</tr>
<tr>
<td></td>
<td>1 pint = 2 cups</td>
</tr>
<tr>
<td></td>
<td>1 cup = 8 fluid ounces</td>
</tr>
</tbody>
</table>

## MASS AND WEIGHT

<table>
<thead>
<tr>
<th>Metric</th>
<th>Customary</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 kilogram = 1000 grams</td>
<td>1 ton = 2000 pounds</td>
</tr>
<tr>
<td>1 gram = 1000 milligrams</td>
<td>1 pound = 16 ounces</td>
</tr>
</tbody>
</table>

## TIME

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1 year = 365 days</td>
<td></td>
</tr>
<tr>
<td>1 year = 12 months</td>
<td></td>
</tr>
<tr>
<td>1 year = 52 weeks</td>
<td></td>
</tr>
<tr>
<td>1 week = 7 days</td>
<td></td>
</tr>
<tr>
<td>1 day = 24 hours</td>
<td></td>
</tr>
<tr>
<td>1 hour = 60 minutes</td>
<td></td>
</tr>
<tr>
<td>1 minute = 60 seconds</td>
<td></td>
</tr>
</tbody>
</table>

Metric and customary rulers can be found on the tear-out Mathematics Chart in the back of this book.
# Grade 7 Mathematics Chart

<table>
<thead>
<tr>
<th>Perimeter</th>
<th>square</th>
<th>$P = 4s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>rectangle</td>
<td>$P = 2l + 2w$ or $P = 2(l + w)$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Circumference</th>
<th>circle</th>
<th>$C = 2\pi r$ or $C = \pi d$</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Area</th>
<th>square</th>
<th>$A = s^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>rectangle</td>
<td>$A = lw$ or $A = bh$</td>
<td></td>
</tr>
<tr>
<td>triangle</td>
<td>$A = \frac{1}{2} bh$ or $A = \frac{bh}{2}$</td>
<td></td>
</tr>
<tr>
<td>trapezoid</td>
<td>$A = \frac{1}{2} (b_1 + b_2)h$ or $A = \frac{(b_1 + b_2)h}{2}$</td>
<td></td>
</tr>
<tr>
<td>circle</td>
<td>$A = \pi r^2$</td>
<td></td>
</tr>
</tbody>
</table>

$B$ represents the Area of the Base of a three-dimensional figure.

<table>
<thead>
<tr>
<th>Volume</th>
<th>cube</th>
<th>$V = s^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>rectangular prism</td>
<td>$V = lwh$ or $V = Bh$</td>
<td></td>
</tr>
<tr>
<td>triangular prism</td>
<td>$V = Bh$</td>
<td></td>
</tr>
<tr>
<td>cylinder</td>
<td>$V = \pi r^2 h$ or $V = Bh$</td>
<td></td>
</tr>
</tbody>
</table>

| Pi | $\pi$ | $\pi \approx 3.14$ or $\pi \approx \frac{22}{7}$ |
Objective 1

The student will demonstrate an understanding of numbers, operations, and quantitative reasoning.

For this objective you should be able to

- represent and use numbers in a variety of equivalent forms; and
- add, subtract, multiply, or divide to solve problems and justify solutions.

What Are Rational Numbers?

Numbers that can be written in the form of a fraction are called rational numbers. Integers, fractions, mixed numbers, percents, and some decimals are rational numbers. For example, the integer 7 can be written as the fraction \( \frac{7}{1} \). The mixed number \( 3\frac{1}{4} \) is equivalent to the fraction \( \frac{13}{4} \). The decimal 0.3 is equal to \( \frac{3}{10} \). Each of these numbers can be written in the form of a fraction. They are all rational numbers.

How Do You Convert Between Different Forms of Rational Numbers?

Given a rational number in one form (integer, fraction, mixed number, percent, or decimal), you should be able to convert it to an equivalent number in any of the other forms. Here are some guidelines for converting rational numbers.

To convert a fraction to a decimal, divide the numerator of the fraction by the denominator.

Rewrite \( \frac{4}{5} \) as an equivalent decimal.

Divide the numerator, 4, by the denominator, 5.

The fraction \( \frac{4}{5} \) is equivalent to the decimal 0.8.

To convert a decimal to a fraction or mixed number, use the place value of the digit farthest to the right of the decimal point as the denominator. Use the digits to the right of the decimal point as the numerator. Use any digits to the left of the decimal point as the whole-number part of a mixed number.
Objective 1

Rewrite 0.381 as a fraction.
The place value of the digit farthest to the right of the decimal point is thousandths.

0.381

Use 1,000 as the denominator.
Use 381 as the numerator.
The decimal 0.381 is equivalent to the fraction \( \frac{381}{1,000} \).

Rewrite 12.07 as a mixed number.
The place value of the digit farthest to the right of the decimal point is hundredths.

12.07

Use 100 as the denominator.
Use 7 as the numerator.
Use 12 as the whole-number part of the mixed number.
The decimal 12.07 is equivalent to the mixed number \( 12\frac{7}{100} \).

To convert a decimal to a percent, multiply by 100. Then place the percent sign after the number.

Rewrite 0.675 as a percent.
Multiply 0.675 by 100. This moves the decimal point two places to the right.

\[ 0.675 \times 100 = 67.5 \]

Write a percent sign.

\[ 0.675 = 67.5\% \]
The decimal 0.675 is equivalent to 67.5%.

To convert a fraction to a percent, first convert the fraction to a decimal by dividing the numerator by the denominator. Then convert the decimal to a percent by multiplying by 100 and writing a percent sign.
A bar over a decimal number indicates a repeating decimal. The fractions $\frac{1}{3}$ and $\frac{2}{3}$ convert to repeating decimals.

$0.\overline{3} = 0.333333\ldots$

$0.\overline{6} = 0.666666\ldots$

**Objective 1**

Rewrite $\frac{5}{6}$ as a percent.
Divide the numerator, 5, by the denominator, 6.

$$5 \div 6 = 0.8\overline{3}$$

Multiply by 100 to convert the decimal to a percent.

$$0.8\overline{3} \times 100 = 83.\overline{3}$$

$0.\overline{8} = 83.\overline{3}\%$

The fraction $\frac{5}{6}$ is equivalent to $83.\overline{3}\%$.

To convert a percent to a decimal, divide the percent by 100 and omit the percent sign.

Rewrite 53.2% as a decimal.
Divide 53.2 by 100. This moves the decimal point two places to the left.

$$53.2 \div 100 = 0.532$$

The value 53.2% is equivalent to the decimal 0.532.

To convert a percent to a fraction, express the percent as a fraction with a denominator of 100. If the percent is greater than 100%, you will have to express it as a mixed number.

Rewrite 48% as a fraction.
Use 48 as the numerator and 100 as the denominator.

$$\frac{48}{100}$$

Simplify the fraction.

$$\frac{48}{100} = \frac{12}{25}$$

The value 48% is equivalent to $\frac{12}{25}$.

Rewrite 225% as a mixed number.
Use 225 as the numerator and 100 as the denominator.

$$\frac{225}{100}$$

Rewrite the improper fraction as a mixed number.
Divide 225 by 100.

$$225 \div 100 = 2 \text{ R}25$$
Here are some common equivalent fractions, decimals, and percents.

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Decimal</th>
<th>Percent</th>
<th>Fraction</th>
<th>Decimal</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{2}$</td>
<td>0.5</td>
<td>50%</td>
<td>$\frac{3}{5}$</td>
<td>0.6</td>
<td>60%</td>
</tr>
<tr>
<td>$\frac{2}{2}$</td>
<td>1.0</td>
<td>100%</td>
<td>$\frac{4}{5}$</td>
<td>0.8</td>
<td>80%</td>
</tr>
<tr>
<td>$\frac{1}{3}$</td>
<td>$0.\overline{3}$</td>
<td>33 $\frac{1}{3}$%</td>
<td>$\frac{5}{5}$</td>
<td>1.0</td>
<td>100%</td>
</tr>
<tr>
<td>$\frac{2}{3}$</td>
<td>$0.\overline{6}$</td>
<td>66 $\frac{2}{3}$%</td>
<td>$\frac{1}{8}$</td>
<td>0.125</td>
<td>12.5%</td>
</tr>
<tr>
<td>$\frac{3}{3}$</td>
<td>1.0</td>
<td>100%</td>
<td>$\frac{2}{8}$</td>
<td>0.25</td>
<td>25%</td>
</tr>
<tr>
<td>$\frac{1}{4}$</td>
<td>0.25</td>
<td>25%</td>
<td>$\frac{3}{8}$</td>
<td>0.375</td>
<td>37.5%</td>
</tr>
<tr>
<td>$\frac{2}{4}$</td>
<td>0.5</td>
<td>50%</td>
<td>$\frac{4}{8}$</td>
<td>0.5</td>
<td>50%</td>
</tr>
<tr>
<td>$\frac{3}{4}$</td>
<td>0.75</td>
<td>75%</td>
<td>$\frac{5}{8}$</td>
<td>0.625</td>
<td>62.5%</td>
</tr>
<tr>
<td>$\frac{4}{4}$</td>
<td>1.0</td>
<td>100%</td>
<td>$\frac{6}{8}$</td>
<td>0.75</td>
<td>75%</td>
</tr>
<tr>
<td>$\frac{1}{5}$</td>
<td>0.2</td>
<td>20%</td>
<td>$\frac{7}{8}$</td>
<td>0.875</td>
<td>87.5%</td>
</tr>
<tr>
<td>$\frac{2}{5}$</td>
<td>0.4</td>
<td>40%</td>
<td>$\frac{8}{8}$</td>
<td>1.0</td>
<td>100%</td>
</tr>
</tbody>
</table>
**Try It**
Sarah answered 22 out of 25 questions correctly on her quiz. What percent of the quiz questions did she answer correctly?

The fraction of questions Sarah answered correctly is \(\frac{22}{25}\).

Convert this fraction to a decimal by dividing \(\frac{22}{25}\) by \(\frac{22}{25}\).

\[
\frac{22}{25} = 0.88
\]

The fraction \(\frac{22}{25}\) is equivalent to the decimal 0.88.

Convert the decimal to a percent by moving the decimal point two places to the right. The decimal 0.88 is equivalent to 88%. Sarah answered 88% of the quiz questions correctly.

At a school football game, 35% of the people watching the game were middle school students. What fraction of the people watching the game were middle school students?

Convert 35% to a decimal. Move the decimal point two places to the left: 35% = 0.35. Rewrite the decimal 0.35 as a fraction: 0.35 = \(\frac{35}{100}\). Simplify the fraction: \(\frac{35}{100} = \frac{7}{20}\). Of the people watching the game, \(\frac{7}{20}\) were middle school students.
How Do You Compare and Order Rational Numbers?

To place a group of rational numbers in order, first convert them to the same form, such as fractions or decimals, and then place them in order. The form you use will depend on the numbers with which you are working.

Here are some guidelines for comparing numbers and placing them in order.

- To compare whole numbers, compare the digits in each place value, starting at the left.
- To compare fractions with the same denominator, compare the numerators.
- To compare fractions with different denominators, rewrite the fractions as equivalent fractions with the same denominator. Then compare the numerators.
- To compare mixed numbers, first compare the whole-number parts. If the whole-number parts are equal, then compare the fractional parts.
- To compare decimal numbers, compare the digits in each place value, starting at the left. Remember that when you compare decimals, the numbers should all have the same number of places to the right of the decimal point. Place zeros to the right of the last digit if necessary.

Some problems ask you to find a number that is between two other numbers. A number is between two numbers if it is greater than one of the numbers and less than the other number.

Of the numbers \(10\frac{1}{3}\), 9, and 10.5, which is between the other two numbers?

- Look at the three numbers plotted on the number line above. The number \(10\frac{1}{3}\) is to the left of 10.5, so \(10\frac{1}{3}\) is less than 10.5. The number 9 is to the left of \(10\frac{1}{3}\), so 9 is less than \(10\frac{1}{3}\). From least to greatest, the numbers are 9, \(10\frac{1}{3}\), and 10.5.
- You can also use symbols (>, <, or =) to represent the relationship between numbers.

\[
10\frac{1}{3} < 10.5 \\
9 < 10\frac{1}{3} \\
9 < 10\frac{1}{3} < 10.5
\]

The number \(10\frac{1}{3}\) is between the numbers 9 and 10.5.
Compare the following numbers. How are these numbers related?

\[ 33\frac{1}{3}, \frac{1}{3}, 2.5, 2\frac{1}{2} \]

- One way to compare the numbers is to convert them all to fractions.

\[ 33\frac{1}{3} = \frac{100}{3}, \frac{1}{3} = \frac{1}{3}, 2.5 = 2\frac{1}{2}, 2\frac{1}{2} = 2\frac{1}{2} \]

- Two of the numbers are equal to \( \frac{1}{3} \).

\[ 33\frac{1}{3} = \frac{1}{3}, \frac{1}{3} = \frac{1}{3} \]

- Two of the numbers are equal to \( 2\frac{1}{2} \).

\[ 2.5 = 2\frac{1}{2}, 2\frac{1}{2} = 2\frac{1}{2} \]

- The numbers that equal \( 2\frac{1}{2} \) are greater than the numbers that equal \( \frac{1}{3} \).

The numbers 2.5 and \( 2\frac{1}{2} \) are greater than \( 33\frac{1}{3} \) and \( \frac{1}{3} \).

In other problems you may be asked to find the greatest or least number in a group of rational numbers.

Which of these three rational numbers is the greatest?

\[ 0.237, \frac{1}{4}, 24\% \]

- Convert all the numbers to decimals so you can compare the place value of the digits. Write zeros as needed so the numbers have the same number of decimal places.

\[ 0.237 = 0.237, \frac{1}{4} = 0.250, 24\% = 0.240 \]  
Divide the numerator, 1, by the denominator, 4.
Move the decimal point two places to the left.

- Compare the decimals.

Look at the ones place. The numbers all have a 0 in the ones place.
Look at the next place, tenths. The numbers all have a 2 in the tenths place.
Look at the next place, hundredths.

\[ 0.237, 0.250, 0.240 \]

Since 5 is the greatest value, 0.25 or \( \frac{1}{4} \) is the greatest number.
Some problems ask you to place a group of rational numbers in order.

Place this list of numbers in order from least to greatest.

\[ 72\% \quad 0.713 \quad \frac{3}{4} \quad \frac{2}{3} \]

- Convert all the numbers to decimals.

\[ 72\% = 0.72 \quad \text{Move the decimal point two places to the left.} \]
\[ 0.713 = 0.713 \]
\[ \frac{3}{4} = 0.75 \quad \text{Divide the numerator, 3, by the denominator, 4.} \]
\[ \frac{2}{3} = 0.666 \quad \text{Divide the numerator, 2, by the denominator, 3.} \]

- Compare the decimals. Write zeros as needed to create the same number of decimal places.

\[ 0.720 \quad 0.713 \quad 0.750 \quad 0.666 \]

Look at the ones place. The numbers all have a 0 in the ones place.

Look at the next place, tenths.

Since 6 is the least value, 0.666 is the least number. List \( \frac{2}{3} \) first.

- The remaining numbers all have a 7 in the tenths place.

Look to the next place, hundredths.

Since \( 1 < 2 < 5 \), the three numbers from least to greatest are

\[ 0.713 \quad 0.720 \quad 0.750 \]

This is the same as:

\[ 0.713 \quad 72\% \quad \frac{3}{4} \]

The four numbers listed in order from least to greatest are as follows:

\[ \frac{2}{3} \quad 0.713 \quad 72\% \quad \frac{3}{4} \]
Try It

The table shows the times it took several students to run a 1-mile race. In what order did the students finish the race?

<table>
<thead>
<tr>
<th>Student</th>
<th>Time (minutes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Barb</td>
<td>7.23</td>
</tr>
<tr>
<td>Gary</td>
<td>7(\frac{1}{2})</td>
</tr>
<tr>
<td>Wallace</td>
<td>8.2</td>
</tr>
<tr>
<td>Tanya</td>
<td>7(\frac{3}{4})</td>
</tr>
</tbody>
</table>

The student with the least time will finish the race __________ .
The student with the greatest time will finish the race __________ .
The greatest number is _______ because _______ is greater than _______ , so it will be the last number in the list.
The other three numbers all have _______ as their whole-number part.
The mixed number 7\(\frac{1}{2}\) expressed as a decimal is _______ .
The mixed number 7\(\frac{3}{4}\) expressed as a decimal is _______ .
Since 2 < _______ < 7, it follows that _______ < _______ < _______ .
The numbers in order from least to greatest are _______ , _______ , _______ , and _______ .
The original numbers in order from least to greatest are _______ , _______ , _______ , and _______ .
The order in which the students finished the race is _______ , _______ , _______ , and _______ .

The student with the least time will finish the race first. The student with the greatest time will finish the race last. The greatest number is 8.2 because 8 is greater than 7. The other three numbers all have 7 as their whole-number part. The number 7\(\frac{1}{2}\) = 7.5, and 7\(\frac{3}{4}\) = 7.75. Since 2 < 5 < 7, it follows that 7.23 < 7.5 < 7.75. The numbers in order from least to greatest are 7.23, 7.5, 7.75, and 8.2. The original numbers in order from least to greatest are 7.23, 7\(\frac{1}{2}\), 7\(\frac{3}{4}\), and 8.2. The order in which the students finished the race is Barb, Gary, Tanya, and Wallace.
How Do You Compare and Order Integers?

Integers are the set of all positive and negative whole numbers and zero. You can represent integers on a number line that extends in both directions from 0.

You can use a number line to compare any two integers, x and y.

- If the integer x is to the left of the integer y on the number line, then \( x < y \).
- If the integer x is to the right of the integer y on the number line, then \( x > y \).

Use the number line below to compare 2, \( -8 \), and \( -3 \).

Since \( -8 \) is to the left of \( -3 \) on the number line, \( -8 < -3 \).
Since \( -3 \) is to the left of 2 on the number line, \( -3 < 2 \).

-8 < -3 < 2

Try It

Use a number line to order \( -4 \), 3, and \( -1 \) from least to greatest.

Since \( _____ \) is to the left of \( -1 \) on the number line, \( _____ < _____ \).

Since \( _____ \) is to the left of 3 on the number line, \( _____ < _____ \).

The numbers in order from least to greatest are \( _____ \), \( _____ \), and \( _____ \).

Since \( -4 \) is to the left of \( -1 \) on the number line, \( -4 < -1 \). Since \( -1 \) is to the left of 3 on the number line, \( -1 < 3 \). The numbers in order from least to greatest are \( -4 \), \( -1 \), and \( 3 \).
What Are the Square of a Number and the Square Root of a Number?

The square of a number is the product of a number and itself. For example, the square of 5 is 25 because $5 \cdot 5 = 25$.

This relationship can also be written as $5^2 = 25$. The expression $5^2 = 25$ is read five squared equals 25.

The figure below illustrates why the term square is used to describe the relationship between 5 and 25. A square with a side of 5 units has an area of 25 square units: 5 squared equals 25.

The square root of a given number is a number that when multiplied by itself equals the given number. For example, the square root of 36 is 6 because $6 \cdot 6 = 36$. The symbol for square root is $\sqrt{\,}$. The square root of 36 is written as $\sqrt{36}$ and is read the square root of 36. The figure below illustrates one way to find the square root of 36.

How can you arrange 36 circles so they form a square?

The only way to do so is to put them in 6 rows, with 6 circles in each row.

The figure shows that $6 \cdot 6 = 36$ and that $\sqrt{36} = 6$.

Here are some more examples of squares and square roots.

$4^2 = 16$, $\sqrt{16} = 4$
$7^2 = 49$, $\sqrt{49} = 7$
$10^2 = 100$, $\sqrt{100} = 10$
Try It
The model below can be used to find the square root of what number?

Each side of the square is _____ units long.
There are _____ unit squares in the large square.
You can use this information to write the equation
_____ • _____ = _____.
This model could be used to find \( \sqrt{____} \).

Each side of the square is 9 units long. There are 81 unit squares in the large square. You can use this information to write the equation \( 9 \cdot 9 = 81 \). This model could be used to find \( \sqrt{81} \).

How Can You Represent Multiplication and Division of Fractions and Decimals?
You can represent multiplication and division of fractions and decimals by using models, pictures, words, and numbers. A model shows the relationship between the decimals or fractions in the problem. The model may be an actual model, such as one made out of blocks, or a picture, an equation, or an expression.

Francis is making cookies. Each cookie needs \( \frac{3}{4} \) teaspoon of colored sprinkles. Francis has \( 13\frac{1}{2} \) teaspoons of colored sprinkles.
Write an expression to find how many cookies he can decorate with this quantity of sprinkles.
• The whole, \( 13\frac{1}{2} \) teaspoons, will be separated into a number of equal smaller parts, each \( \frac{3}{4} \) teaspoon.
• Use division to separate a whole into equal parts.
The expression \( 13\frac{1}{2} \div \frac{3}{4} \) represents how many cookies Francis can decorate with \( 13\frac{1}{2} \) teaspoons of sprinkles.
How Do You Multiply and Divide Fractions and Decimals?

When you multiply fractions, first multiply the numerators to get the numerator of the product. Then multiply the denominators to get the denominator of the product. The fractions do not need a common denominator.

\[
\frac{9}{10} \cdot \frac{4}{5} = \frac{9 \cdot 4}{10 \cdot 5} = \frac{36}{50}
\]

Simplify your answer.

\[
\frac{36}{50} = \frac{18}{25}
\]

When you multiply mixed numbers, first convert each mixed number to a fraction. Then multiply the fractions using the method above.

Try It

Zena estimates that she needs \(3\frac{1}{2}\) yards of fabric to make a new dress. The fabric she wants costs $8.75 per yard. What expression can Zena use to find the cost of the fabric needed to make the dress?

Use the operation of \(\underline{\text{____________________}}\) to find the total cost of the fabric.

Convert \(3\frac{1}{2}\) to a decimal: \(3\frac{1}{2} = \underline{______}\).

The expression \(\underline{______} \cdot \underline{______}\) can be used to find the total cost of the fabric.

Use the operation of \underline{multiplication} to find the total cost of the fabric. The number \(3\frac{1}{2} = 3.5\).

The expression $8.75 \cdot 3.5$ can be used to find the total cost of the fabric.
Reciprocals are numbers whose products are equal to 1. The numbers \( \frac{4}{3} \) and \( \frac{3}{4} \) are reciprocals because \( \frac{4}{3} \cdot \frac{3}{4} = \frac{12}{12} = 1 \).

How many \( \frac{2}{3} \)-cup servings of punch can be obtained from 4 cups of punch?

- Use the operation of division to divide a whole (4 cups) into equal parts (\( \frac{2}{3} \) cup).
  
  \[ 4 \div \frac{2}{3} \quad \text{The fraction} \frac{2}{3} \text{ is the divisor.} \]

- To divide, multiply by the reciprocal of the divisor.
  
  \[ 4 \div \frac{2}{3} = 4 \cdot \frac{3}{2} \quad \text{The reciprocal of} \frac{2}{3} \text{ is} \frac{3}{2}. \]

- The whole number 4 is the same as the fraction \( \frac{4}{1} \).
  
  \[ \frac{4}{1} \cdot \frac{3}{2} = \frac{4 \cdot 3}{1 \cdot 2} = \frac{12}{2} = \frac{6}{1} = 6 \]

There are 6 servings of \( \frac{2}{3} \) cup each in 4 cups of punch.

To divide a number by a fraction, multiply the number by the reciprocal of the divisor.

Multiply \( \frac{2}{3} \) by \( \frac{1}{3} \):
- Convert the mixed number \( \frac{2}{3} \) to a fraction.
  
  \[ \frac{2}{3} = \frac{(7 \cdot 3) + 2}{7} = \frac{23}{7} \]

- Multiply the fractions.
  
  \[ \frac{23}{7} \cdot \frac{1}{3} = \frac{23 \cdot 1}{7 \cdot 3} = \frac{23}{21} \]

- Convert the answer to a mixed number.
  
  \[ \frac{23}{21} = 23 \div 21 = 1 \text{ R}2 \]

- Express the remainder as a fraction. Use the remainder, 2, as the numerator. Use the divisor, 21, as the denominator.
  
  \[ 1 \text{ R}2 = 1 \frac{2}{21} \]

\[ \frac{2}{3} \cdot \frac{1}{3} = 1 \frac{2}{21} \]
**Try It**

Divide $\frac{5}{12}$ by $\frac{5}{8}$.

The fraction $\frac{5}{12}$ is the divisor.

The reciprocal of $\frac{5}{8}$ is $\frac{8}{5}$.

Multiply $\frac{5}{12}$ by the reciprocal of the divisor.

$$\frac{5}{12} \div \frac{5}{8} = \frac{5}{12} \cdot \frac{8}{5} = \frac{5 \cdot 8}{12 \cdot 5} = \frac{40}{60} = \frac{2}{3}$$

The fraction $\frac{5}{8}$ is the divisor. The reciprocal of $\frac{5}{8}$ is $\frac{8}{5}$. Multiply $\frac{5}{12}$ by the reciprocal of the divisor.

$$\frac{5}{12} \div \frac{5}{8} = \frac{5}{12} \cdot \frac{8}{5} = \frac{5 \cdot 8}{12 \cdot 5} = \frac{40}{60} = \frac{2}{3}$$
To multiply decimals, first multiply the numbers as if they were whole numbers. Then place the decimal point correctly in the product. You do not need to line up the decimal points in multiplication as you do in addition and subtraction.

Before you multiply decimals, you could round to the nearest whole number to estimate the answer. This is a good way to be certain you have placed the decimal point correctly in the product.

Multiply 2.14 by 3.2.

- Estimate the answer by rounding: $2.14 \cdot 3.2$ is approximately $2 \cdot 3$, which is 6. Your answer should be close to 6.
- Multiply the numbers.

\[
\begin{array}{c}
2.14 & \text{Factor} \\
\times & 3.2 & \text{Factor} \\
\hline
428 & \\
+ 6420 & \\
\hline
6848 & \text{Product}
\end{array}
\]

- To place the decimal point correctly in the answer, count the number of decimal places to the right of the decimal point in each of the factors.

2.14 2 decimal places to the right of the decimal point
3.2 1 decimal place to the right of the decimal point

Since there are a total of 3 decimal places to the right of the decimal points in the two factors, there should be 3 decimal places to the right of the decimal point in the product.

\[6.848\]

- Compare 6.848 to your estimate, which was about 6. Since 6.848 is close to 6, you placed the decimal point correctly in the product.

\[2.14 \cdot 3.2 = 6.848\]
To divide a number by a decimal, count the number of decimal places to the right of the decimal point in the divisor. Then move the decimal point to the right that many places in both the divisor and the dividend. Divide as you would for whole numbers. Then place the decimal point in the correct place in the quotient.

Before you divide decimals, you could round to the nearest whole number to estimate the answer. This is a good way to be certain you have placed the decimal point in the correct place in the quotient.

Divide 9.12 by 2.4.

- Estimate the answer by rounding: $9.12 \div 2.4$ is approximately $9 \div 2$, which is about 4. Your answer should be close to 4.

- Count the number of decimal places to the right of the decimal point in the divisor. In this case, it is one place. Move the decimal points and divide.

\[
\begin{array}{c|c|c}
\text{Divisor} & 2.4 & \text{Dividend} \\
\hline
9.12 & 2.4 & 91.2 \\
\hline
24 & 3.8 & 91.2 \\
\hline
& -72 & \text{Move the decimal one place to the right in the divisor and the dividend. Then move the decimal point in the dividend straight up into the quotient. Divide.} \\
\hline
& 192 & \\
\hline
& -192 & \\
\hline
& 0 & \\
\end{array}
\]

- Compare 3.8 to your estimate, which was about 4. Since 3.8 is close to 4, you placed the decimal point correctly in the quotient.

$9.12 \div 2.4 = 3.8$
Divide 4.5 by 0.75.

- Estimate the answer by rounding: 4.5 ÷ 0.75 is approximately 5 ÷ 1, which is equal to 5. Your answer should be about 5.
- Count the number of decimal places to the right of the decimal point in the divisor. In this case, it is two places. Move the decimal points and divide.

\[
\begin{array}{c}
\text{Divisor} \quad 0.75 \\
\text{Quotient} \quad 4.50 \\
\text{Dividend}
\end{array}
\]

Move the decimal two places to the right in the divisor and the dividend. Since 4.5 has only one digit to the right of the decimal point, place a 0 to the right of the 5. Then move the decimal point in the dividend straight up into the quotient. Divide.

\[
\begin{array}{c}
75 \\
450 \\
450 \\
0
\end{array}
\]

- Compare 6 to your estimate, which was about 5. Since 6 is close to 5, you placed the decimal point correctly in the quotient.

\[4.5 \div 0.75 = 6\]

---

**Try It**

Multiply 15 by 4.12.

Estimate the answer. Your answer should be about

\[____ \cdot ____ , \text{ or } ____ .\]

Multiply the numbers.

\[
\begin{array}{c}
4.12 \\
\times \quad 15
\end{array}
\]

Place the decimal point correctly in the answer. There are a total of ______ decimal places to the right of the decimal point in the two factors.

The product of 15 and 4.12 is ______ .

\[
\begin{array}{c}
4.12 \\
\times \quad 15
\end{array}
\]

\[
\begin{array}{c}
2060 \\
4120 \\
61.80
\end{array}
\]

Your answer should be about 15 • 4, or 60. There are a total of two decimal places to the right of the decimal point in the two factors. The product of 15 and 4.12 is 61.80.
**Try It**
Divide 55 by 2.5.

Estimate the answer. The problem $55 \div 2.5$ is approximately $60 \div \underline{______}$, which is about 20.

Set up the problem.

\[
\begin{array}{c}
\frac{55}{2.5} \div \underline{______}
\end{array}
\]

There is/are ______ digit(s) to the right of the decimal in the divisor, 2.5. Move the decimal in the divisor and the dividend ______ place(s) to the ______. In order to move the decimal in the dividend, place a ______ after the 5 in the ones place.

\[
\frac{55}{2.5} = \underline{______}
\]

The problem $55 \div 2.5$ is approximately $60 \div 3$, which is ______.

There is one digit to the right of the decimal in the divisor, 2.5. Move the decimal in the divisor and the dividend one place to the right. In order to move the decimal in the dividend, place a 0 after the 5 in the ones place.

\[
\begin{align*}
55 & \div 2.5 = 22 \\
25 & \left\{ \begin{array}{c}
\begin{array}{c}
50 \\
50 \end{array}
\end{array} \right.
\end{align*}
\]
How Do You Solve Problems Involving Fractions and Decimals?

You solve problems that involve fractions and decimals using the same steps as in any other problem. First make sure you understand the problem. Identify the quantities involved and the relationships between them. Write an equation that can be used to find the answer. Solve the equation and then check your answer to see if it is reasonable.

Troy has 9 cans of fish. If each can holds 12.5 ounces, how many ounces of canned fish does Troy have in all?

- There are 9 cans of fish. Each can holds 12.5 ounces.
  
  Use multiplication to combine equal groups.
  
  The total ounces of canned fish equals 9 \cdot 12.5.

\[
\begin{align*}
12.5 \\
\times 9 \\
\hline
112.5
\end{align*}
\]

There is one decimal place to the right of the decimal point in the two factors. So there will be one decimal place to the right of the decimal point in the product. The product of 9 and 12.5 is 112.5.

- Check to see if your answer is reasonable.
  
  Since 9 \cdot 12.5 is close to 120, the product of 10 and 12, the answer 112.5 seems reasonable.

Troy has a total of 112.5 ounces of canned fish.
**Try It**

Molly has $3\frac{1}{4}$ pounds of flour in a container. She also has a bag with 2.5 pounds of flour in it. If Molly pours half the flour from the bag into the container, how much flour will be in the container?

Use the operation of \(\frac{1}{2}\) to separate the flour in the bag into two equal parts.

\[
\frac{2.5}{2} = 1.25
\]

Use the operation of addition to find how much flour will then be in the container.

The flour in the container can be represented by the expression

\[
\frac{3}{4} + 1.25
\]

Convert $3\frac{1}{4}$ to an equivalent decimal: $3\frac{1}{4} = 3.25$.

\[
3.25 + 1.25 = 4.5
\]

The container will have 4.5 pounds of flour in it.

---

**How Can You Model Addition, Subtraction, Multiplication, and Division of Integers?**

You can model addition and subtraction of integers using a number line. These operations can be modeled with an arithmetic expression.

Use the number line below to model the addition problem $-6 + 2$.

- Start to model the addition problem at 0.
- Since $-6$ is a negative integer, it can be modeled with an arrow 6 units to the left of 0.
  - Draw an arrow from 0 to $-6$, which is 6 units to the left of 0.
- Since 2 is a positive integer, it can be modeled with an arrow 2 units to the right of $-6$.
  - Draw an arrow from $-6$ to $-4$, which is 2 units to the right of $-6$.

The model shows that the sum of $-6$ and 2 is $-4$. 
Try It
Linda scored 14 points in a board game. On her next turn she lost 6 points, and then she lost 5 more points. On her final turn Linda gained 8 points. Determine Linda’s final score by modeling the problem on a number line.

Start at 0 on the number line. Linda’s beginning score is 14. Represent this score with the integer _______. To add 14, count up 14 places from 0.

On her next turn Linda lost 6 points. Represent this with the integer _______. To add −6, count back 6 places from 14 to _______.

On her next turn Linda lost 5 points. Represent this with the integer _______. To add −5, count back 5 places from 8 to _______.

On her final turn Linda scored 8 points. Represent this with the integer _______. To add 8, count up 8 places from 3 to _______.

Represent Linda’s final score with the expression _______ + _______ + _______ + _______.

Linda’s final score was _______.

Linda’s beginning score is 14. Represent this score with the integer 14. On her next turn Linda lost 6 points. Represent this with the integer −6. To add −6, count back 6 places from 14 to 8. On her next turn Linda lost 5 points. Represent this with the integer −5. To add −5, count back 5 places from 8 to 3. On her final turn Linda scored 8 points. Represent this with the integer 8. To add 8, count up 8 places from 3 to 11. Represent Linda’s final score with the expression 14 + (−6) + (−5) + 8. Linda’s final score was 11.
Some problems may require you to multiply and divide integers.

Gloria had $123 in her savings account in January. During the year she made 3 deposits of $25 each and 2 withdrawals of $30 each. Write an expression to represent the total amount of money Gloria had in the bank after her deposits and withdrawals.

The expression $3 \cdot 25$ represents the total amount deposited. The expression $2 \cdot 30$ represents the total amount withdrawn.

The expression $123 + (3 \cdot 25) - (2 \cdot 30)$ represents the total amount of money Gloria had in the bank after her deposits and withdrawals.

Try It

Mr. Taylor purchased hamburger patties and buns for a party. Each package of hamburger patties contained 10 patties. Each package of buns contained 8 buns. Mr. Taylor purchased 9 packages of patties and 11 packages of buns. What expression could be used to represent how many more hamburger patties Mr. Taylor had than buns?

Each package of hamburger patties contained ______ patties.

Mr. Taylor purchased 9 packages of patties. Use the operation of _______•_______ to find the total number of patties.

Each package of buns contained ______ buns. Mr. Taylor purchased 11 packages of buns. Use the operation of _______•_______ to find the total number of buns.

Use the operation of _______•_______ to find the difference between the number of hamburger patties and the number of buns.

The following expression represents how many more hamburger patties Mr. Taylor had than buns.

$$ (9 \cdot 10) - (11 \cdot 8) $$

Each package of hamburger patties contained 10 patties. Use multiplication to find the total number of patties: $9 \cdot 10$. Each package of buns contained 8 buns. Use multiplication to find the total number of buns: $11 \cdot 8$. Use subtraction to find the difference between the number of hamburger patties and the number of buns. The following expression represents how many more hamburger patties Mr. Taylor had than buns: $9 \cdot 10 - 11 \cdot 8$. 

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How Can You Use a Unit Rate to Solve Problems?

Some problems can be solved by expressing the ratio of quantities in the problem as a unit rate, which is a rate with a denominator of 1.

A ratio is a comparison of two quantities. For example, if you go to school 5 days out of every week, the ratio of days at school to days in the week is \( \frac{5}{7} \).

A rate is also a ratio. If you use 2 gallons of gasoline to drive 42 miles, then the rate at which you are using gasoline is \( \frac{42 \text{ miles}}{2 \text{ gallons}} \), or \( \frac{42}{2} \).

A unit rate is a rate with a denominator of 1. In the example above, the rate at which you are using gasoline, \( \frac{42}{2} \), can be simplified.

\[
\frac{\text{miles}}{\text{gallons}} = \frac{42}{2} = \frac{21}{1}
\]

The unit rate is 21 miles to 1 gallon, 21:1, or 21 miles per 1 gallon.

Unit rates are usually written without the 1, such as 21 miles per gallon. Unit rates can be helpful in comparing prices or working with recipes.

Martha is baking pies. Her recipe calls for 3 cups of flour to make 2 pie crusts. How many cups of flour will Martha use per pie crust?

- First write a ratio that compares the number of cups of flour to the number of pie crusts.

\[
\frac{\text{cups of flour}}{\text{pie crusts}} = \frac{3}{2}
\]

This shows a rate of 3 cups of flour to every 2 pie crusts.

- Divide the numerator and denominator by 2 to find the unit rate.

\[
\frac{\text{cups of flour}}{\text{pie crusts}} = \frac{3}{2} = \frac{1.5}{1}
\]

Martha will use 1.5 cups of flour per pie crust.
Rae needs a piece of chain to lock her gate. The hardware store sells 3-foot lengths of chain for $6.30 each. Use a unit rate to find the cost of the chain per foot.

- First write a ratio that compares the total cost to the length of the chain.

\[
\frac{\text{total cost}}{\text{length of chain}} = \frac{\$6.30}{3 \text{ ft}}
\]

- Divide the numerator and denominator by 3 to find the unit rate.

\[
\frac{\text{total cost}}{\text{length of chain}} = \frac{\$6.30}{3 \text{ ft}} = \frac{\$2.10}{1 \text{ ft}}
\]

The cost of the chain is $2.10 per foot.

**Try It**

Joe and his cousin are driving from Houston, Texas, to Panama City, Florida. After 11 hours they have driven 605 miles. What is their average driving speed in miles per hour?

Write a ratio that compares the number of miles Joe and his cousin have driven to the number of hours they have driven.

\[
\frac{\text{miles}}{\text{hours}} = \frac{605}{11}
\]

Their average driving speed is a _____ rate. To find this rate, divide the numerator and denominator by _____.

\[
\frac{\text{miles}}{\text{hours}} = \frac{605}{11} = \frac{55}{1}
\]

Joe and his cousin are driving at an average speed of _____ miles per hour.

The ratio is \(\frac{605}{11}\). Their average driving speed is a unit rate. To find this rate, divide the numerator and denominator by 11: \(\frac{605}{11} = \frac{55}{1}\). Joe and his cousin are driving at an average speed of 55 miles per hour.
How Do You Use the Order of Operations to Simplify Numerical Expressions?

Many calculations require you to perform several arithmetic operations to reach a solution. There are rules for the order in which these operations are performed.

To simplify an expression, follow this order of operations:

1. Simplify any operations in parentheses and brackets. If there is more than one operation within a set of parentheses, follow the order of operations in steps 2, 3, and 4.
2. Simplify any terms with exponents.
3. Do all multiplication and division from left to right.
4. Do all addition and subtraction from left to right.

Simplify the expression $3 \cdot 2 + 4 \cdot 5 - 4 \cdot 2$.
- There are no parentheses or exponents.
- Begin at the left and multiply from left to right.

$$3 \cdot 2 + 4 \cdot 5 - 4 \cdot 2 = 6 + 20 - 8$$
- Then begin at the left and add or subtract from left to right.

$$6 + 20 - 8 = 26 - 8 = 18$$

The expression $3 \cdot 2 + 4 \cdot 5 - 4 \cdot 2$ simplifies to 18.

Simplify the expression $4 + (5 \cdot 2 - 1) \div 3 \cdot 2^2$.
- First perform any operations in parentheses. There are multiplication and subtraction signs within the parentheses. The order of operations says to multiply before you subtract.

$$4 + (5 \cdot 2 - 1) \div 3 \cdot 2^2 = 4 + (10 - 1) \div 3 \cdot 2^2 = 4 + 9 \div 3 \cdot 2^2$$
- Then simplify any expressions with exponents.

$$4 + 9 \div 3 \cdot 2^2 = 4 + 9 \div 3 \cdot 4$$
Try It
Simplify the expression \((5 \cdot 7 + 3) - 8 \div 4\).

Follow the order of operations.

\[
(5 \cdot 7 + 3) - 8 \div 4 = \\
(____ + 3) - 8 \div 4 = \\
_____ - 8 \div 4 = \\
_____ - _____ = \\
_____
\]

\((5 \cdot 7 + 3) - 8 \div 4 = _____\)

\[
(5 \cdot 7 + 3) - 8 \div 4 = \\
(35 + 3) - 8 \div 4 = \\
38 - 8 \div 4 = \\
38 - 2 = \\
36 \\
(5 \cdot 7 + 3) - 8 \div 4 = 36
\]
Try It
Simplify the expression $1 + 8 \div 2 \cdot 3 - 1 + 3^3$.
Follow the order of operations.

\[
1 + 8 \div 2 \cdot 3 - 1 + 3^3 =
\]

\[
1 + 8 \div 2 \cdot 3 - 1 + \underline{______} =
\]

\[
1 + \underline{______} \cdot 3 - 1 + \underline{______} =
\]

\[
1 + \underline{______} - 1 + 27 =
\]

\[
\underline{______} - 1 + 27 =
\]

\[
\underline{______} + 27 =
\]

\[
1 + 8 \div 2 \cdot 3 - 1 + 3^3 = \underline{______}
\]

How Do You Determine Whether a Solution to a Problem Is Reasonable?

One way to determine whether an answer to a problem is reasonable (makes sense) is to round the numbers in the problem before you do the arithmetic. Then solve the problem using the rounded numbers. When you do this, you are making an estimate of the answer. The estimate tells you about how big or small the exact answer should be. It is always a good idea to estimate first. Then you will know whether your exact answer is reasonable.

You can also estimate when you do not need an exact answer to a problem. For example, some problems ask about how many or approximately how much. Use estimation to solve such problems.
The sticker on Mr. Hart’s new car shows that its average gas mileage is 29 miles per gallon of gas. Mr. Hart drives about 87 miles each day. About how much gas does Mr. Hart use in 12 days of driving?

- Estimate how many miles Mr. Hart drives in 12 days.
  
  \[ 87 \text{ miles per day} \times 12 \text{ days} = \text{number of miles Mr. Hart drives} \]
  
  Round 87 to 90, and round 12 to 10.
  
  \[ 90 \times 10 = 900 \]

  Mr. Hart drives about 900 miles in 12 days.

- Estimate how much gas Mr. Hart will use to drive 900 miles.
  
  \[ \frac{900 \text{ miles}}{29 \text{ miles per gallon}} = \text{number of gallons of gas Mr. Hart uses} \]
  
  Round 29 to 30.
  
  \[ \frac{900}{30} = 30 \]

  Mr. Hart uses about 30 gallons of gas in 12 days.

**Try It**

A high school yearbook adviser knows that last year about 74% of graduating students purchased a yearbook. This year’s graduating class has 233 students. Based on last year’s purchase rate, the yearbook adviser estimates that 172 graduating students will purchase a yearbook this year. Is this a reasonable estimate?

The number 233 rounded to the nearest 10 is _______.

The value 74% is between 70% and 80%. Use these values to estimate the range in which the answer should lie.

\[ 70\% \text{ of } 230 = 0.70 \times 230 = \ldots \]

\[ 80\% \text{ of } 230 = \ldots \times \ldots = \ldots \]

The number of students who will purchase a yearbook this year is between _______ and _______.

Is 172 students a reasonable estimate? _______.

The number 233 rounded to the nearest 10 is 230. 

70% of 230 = 0.70 \times 230 = 161. 80% of 230 = 0.80 \times 230 = 184. The number of students who will purchase a yearbook this year is between 161 and 184. Is 172 students a reasonable estimate? Yes.
**Question 1**
Bob tried to use an \(\frac{11}{16}\)-inch wrench to remove a bolt, but the wrench was too small. Which size wrench might be large enough to remove the bolt?

- A \(\frac{5}{8}\) in.
- B \(\frac{7}{16}\) in.
- C \(\frac{3}{4}\) in.
- D \(\frac{1}{2}\) in.

**Question 2**
In Tim’s class, 15 of the 25 students have traveled outside their home state. Which number does NOT represent the part of the class that has traveled outside their home state?

- A \(\frac{10}{15}\)
- B 60%
- C 0.6
- D \(\frac{3}{5}\)

**Question 3**
Look at the model below. Which value can this model be used to find?

- A \(\sqrt{16}\)
- B \(\sqrt{32}\)
- C \(\sqrt{8}\)
- D \(\sqrt{64}\)

**Question 4**
Which expression shows how many \(\frac{3}{8}\)-pound hamburger patties can be made from \(4\frac{1}{3}\) pounds of ground beef?

- A \(4\frac{1}{3} \cdot \frac{3}{8}\)
- B \(4\frac{1}{3} ÷ \frac{3}{8}\)
- C \(\frac{3}{8} \cdot \frac{1}{3}\)
- D \(\frac{3}{8} ÷ \frac{1}{3}\)
**Question 5**
Tammy, María, and their three brothers own a business. Tammy owns $\frac{1}{3}$ of the business. María and their three brothers equally share ownership of the remaining part of the business. What fraction of the business does María own?

A $\frac{1}{6}$  
B $\frac{1}{4}$  
C $\frac{1}{2}$  
D $\frac{1}{12}$

**Question 6**
The low temperature recorded one morning was $-5^\circ F$. The high temperature recorded that afternoon was $41^\circ F$. What was the change in temperature from the morning low to the afternoon high?

A $+36^\circ F$  
B $+46^\circ F$  
C $-36^\circ F$  
D $-46^\circ F$

**Question 7**
Glenn is buying 2 pounds of cheese for $7.50. Which expression could be used to represent the cost of 4.5 pounds of cheese?

A $4.5 + 2 \cdot 7.50$  
B $4.5 \cdot \frac{2}{7.50}$  
C $7.50 \div \frac{4.5}{2}$  
D $7.50 \div 2 \cdot 4.5$

**Question 8**
Which of the following should be performed first to simplify this expression?

$$5 + 3 \cdot 2 \div 3^2 - 12$$

A $3 \cdot 2$  
B $5 + 3$  
C $3^3$  
D $2 \div 3$
Question 9
Peter walked at a rate of 4 miles in 50 minutes. Jan walked at a rate of 3 miles in 30 minutes. Which statement correctly describes this situation?

A Jan’s walking rate was 0.18 mile per minute faster than Peter’s.
B Jan’s walking rate was 0.18 mile per minute slower than Peter’s.
C Peter’s walking rate was 0.02 mile per minute faster than Jan’s.
D Peter’s walking rate was 0.02 mile per minute slower than Jan’s.

Question 11
Look at the fractions below.
\[
\frac{13}{16}, \frac{5}{8}, \frac{3}{4}, \frac{3}{8}, \frac{7}{16}
\]
Which lists these fractions from least to greatest?

A \(\frac{7}{8}, \frac{13}{16}, \frac{3}{4}, \frac{5}{8}, \frac{3}{16}\)
B \(\frac{3}{4}, \frac{5}{8}, \frac{3}{16}, \frac{13}{16}, \frac{7}{8}\)
C \(\frac{7}{8}, \frac{3}{4}, \frac{13}{16}, \frac{3}{16}, \frac{7}{8}\)
D \(\frac{3}{16}, \frac{5}{8}, \frac{3}{4}, \frac{13}{16}, \frac{7}{8}\)
Question 12
Which set of grids below best models the number sentence $\frac{3}{4} \cdot \frac{1}{5} = \frac{3}{20}$?

A

B

C

D

Answer Key: page 146
For this objective you should be able to

- solve problems involving proportional relationships;
- represent a relationship in numerical, geometric, verbal, and symbolic form; and
- use equations to solve problems.

**What Are Ratios and Proportions?**

A **ratio** is a comparison of two quantities. For example, a recipe might call for 2 cups of flour for every 1 cup of milk. You can compare the number of cups of flour used to the number of cups of milk used with the ratio two to one. This ratio can be written in several different forms:

\[
\frac{2}{1} \quad 2 \text{ to } 1 \quad 2:1
\]

A **proportion** is a statement that two ratios are equal. For example, the ratio 2 to 1 is the same as the ratio 6 to 3. This can be written as 2:1 = 6:3. It can also be written as follows:

\[
\frac{2}{1} = \frac{6}{3}
\]

If two ratios form a proportion, then their cross products are equal. **Cross products** are the product of the numerator of the first fraction and the denominator of the second fraction and the product of the denominator of the first fraction and the numerator of the second fraction.

\[
\begin{align*}
5 \times 12 &= 6 \times 10 \\
60 &= 60
\end{align*}
\]

**Compare the ratios 2:5 and 6:15. Do they form a proportion?**

- Write the ratios as fractions.
  \[
  \frac{2}{5} \text{ and } \frac{6}{15}
  \]

- Cross products of proportions are equal. Use the cross products to determine whether \( \frac{2}{5} \) and \( \frac{6}{15} \) form a proportion.

  \[
  \begin{align*}
  2 \times 15 &= 5 \times 6 \\
  30 &= 30
  \end{align*}
  \]

Since the cross products are equal, \( \frac{2}{5} \) and \( \frac{6}{15} \) form a proportion.
A variable is a symbol used to represent a number. Letters of the alphabet are frequently used as variables.

Find the value of $k$ that makes the following proportion true.

$$\frac{k}{10.5} = \frac{1.5}{3.5}$$

Set the fractions equal.

$$3.5k = 1.5 \cdot 10.5$$

Use cross products.

$$\frac{3.5k}{3.5} = \frac{15.75}{3.5}$$

Divide both sides of the equation by 3.5.

$$k = 4.5$$

The value for $k$ that makes the proportion true is 4.5.

Many real-life problems can be solved using proportions.

If a 5-pound bag of rice costs $3, how much would a 25-pound bag of rice cost at that rate?

- Identify the quantities being compared.

<table>
<thead>
<tr>
<th>Pounds of Rice</th>
<th>Cost of Rice</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 pounds</td>
<td>$3</td>
</tr>
<tr>
<td>25 pounds</td>
<td>$x</td>
</tr>
</tbody>
</table>

- Write a proportion. Let $x$ equal the cost of 25 pounds of rice.

$$\frac{\text{pounds}}{\text{cost}} = \frac{5}{3} = \frac{25}{x}$$

- Solve the proportion.

$$\frac{5}{3} = \frac{25}{x}$$

Use cross products.

$$5x = 25 \cdot 3$$

Divide both sides of the equation by 5.

$$\frac{5x}{5} = \frac{75}{5}$$

$$x = 15$$

A 25-pound bag of rice would cost $15.
Try It

Gina has been present at school for 42 of the first 45 days of the school year. The school year is 180 days long. If Gina continues to attend school at the same rate, how many days, \( d \), will she attend school during the school year?

Identify the quantities being compared:

<table>
<thead>
<tr>
<th>______ days present in the first ______ days of school, and total days present during the school year, ( d ), in ______ days in the school year.</th>
</tr>
</thead>
</table>

Write a proportion. Remember that \( d \) equals the total number of days Gina will attend school.

\[
\frac{\text{days present}}{\text{days of school}} = \frac{d}{\text{total days}}
\]

To solve the proportion, first use cross products.

\[
\text{_____} \cdot \text{_____} = \text{_____} \cdot \text{_____}
\]

Then divide both sides of the equation by ______.

\[
d = 
\]

At this rate, Gina will attend school ______ days during the school year.

Compare: 42 days present in the first 45 days of school, and total days present during the school year, \( d \), in 180 days in the school year.

The proportion is \( \frac{42}{45} = \frac{d}{180} \). To solve the proportion, first use cross products:

\[
42 \cdot 180 = 45 \cdot d
\]

Then divide both sides of the equation by 45: \( d = 168 \).

At this rate, Gina will attend school 168 days during the school year.
How Do You Solve Problems Involving Percent?
You can use proportions or decimals to solve problems involving percent.

Larry scored 80% on a test with 60 questions. How many questions did he answer correctly?

**Proportion Method**
- Let $x$ represent the number of questions Larry answered correctly.
- Set up a proportion using $\frac{80}{100}$ for 80%.

\[
\frac{\text{part}}{\text{whole}} = \frac{80}{100} = \frac{x}{60}
\]

- Solve the proportion.

\[
\frac{80}{100} = \frac{x}{60} \\
4,800 = 100x \quad \text{Use cross products.} \\
\frac{4,800}{100} = \frac{100x}{100} \\
48 = x
\]

Larry answered 48 questions correctly.

**Decimal Method**
- Convert 80% to a decimal.

\[
80\% = \frac{80}{100} = 0.80
\]

- Multiply 0.80 by 60, the total number of questions.

\[
0.80 \cdot 60 = 48
\]

Larry answered 48 questions correctly.

A survey of 250 people showed that 105 of them had never seen the ocean. What percent of the people surveyed had never seen the ocean?

- Express the ratio 105 to 250 as a fraction.

\[
\frac{105}{250}
\]

- Remember that percent means per 100. Use the fraction $\frac{n}{100}$ to represent the percent. Write a proportion.

\[
\frac{105}{250} = \frac{n}{100}
\]

- Solve for $n$.

\[
\frac{105}{250} = \frac{n}{100} \\
250n = 10,500 \quad \text{Use cross products.} \\
\frac{250n}{250} = \frac{10,500}{250} \\
\frac{n}{42} = \frac{42}{250}
\]

Of the people surveyed, 42% had never seen the ocean.
An office-supply store has school supplies on sale for 20% off the original price. If Allie buys supplies that had an original price of $45.60, how much will she actually pay for the school supplies?

- First find the discount: 20% of $45.60.
  
  Convert 20% to a decimal.
  
  \[
  20\% = \frac{20}{100} = 0.20
  \]
  
  Multiply 0.20 by 45.60, the original price.
  
  \[
  0.20 \times 45.60 = 9.12
  \]
  
  Allie’s discount is $9.12.

- Then find what Allie will pay after the discount.
  
  Subtract the discount from the original price.
  
  \[
  45.60 - 9.12 = 36.48
  \]
  
  Allie will pay $36.48 for the school supplies.

### Try It

In a science experiment 12.5 grams of salt crystals were left out in an open dish. After 24 hours the total mass of the crystals had increased by 3%. What was the mass in grams of the salt crystals after 24 hours?

To find the number of grams by which the mass increased, find 3% of _______.

Convert _______ % to a decimal.

\[
 \text{_______} \% = \text{_______}
\]

Multiply _______ by _______ grams.

\[
 \text{_______} \times \text{_______} = \text{_______}
\]

The mass of the salt crystals increased by _______ gram.

To find the mass after 24 hours, add the increase in mass to the original mass.

\[
 \text{_______} + \text{_______} = \text{_______}
\]

The mass of the salt crystals after 24 hours was _______ grams.

To find the number of grams by which the mass increased, find 3% of 12.5. Convert 3% to a decimal: 3% = 0.03. Multiply 0.03 by 12.5 grams:

\[
 0.03 \times 12.5 = 0.375
\]

The mass of the salt crystals increased by 0.375 gram, and

\[
 12.5 + 0.375 = 12.875
\]

The mass of the salt crystals after 24 hours was 12.875 grams.
How Are Proportions Used to Solve Problems Involving Similar Figures?

Another type of real-life problem that can be solved using a proportion is a problem involving similar figures. **Similar figures** are geometric figures that have the same shape but not necessarily the same size. In similar figures the ratios of the lengths of corresponding sides are proportional. Corresponding sides are sides that are in the same relative position in the two figures.

Rectangle $ABCD$ is similar to rectangle $MNPR$. What is the length of side $BC$ in rectangle $ABCD$?

- Corresponding sides of similar figures are sides that are in the same relative position.
  - $\overline{AB}$ corresponds to $\overline{MN}$
  - $\overline{BC}$ corresponds to $\overline{NP}$
  - $\overline{DC}$ corresponds to $\overline{RP}$
  - $\overline{AD}$ corresponds to $\overline{MR}$

- If two figures are similar, then the ratios of corresponding sides form a proportion.
  - $\frac{\text{large}}{\text{small}} = \frac{\overline{AB}}{\overline{MN}} = \frac{\overline{BC}}{\overline{NP}}$

- Substitute the lengths of the sides in the proportion.
  - $\frac{12}{4} = \frac{x}{6}$

- To find the value of $x$, solve the proportion.
  - $\frac{12}{4} = \frac{x}{6}$
  - $4x = 72$ (Use cross products.)
  - $x = 18$ (Divide both sides of the equation by 4.)

The length of side $BC$ is 18 units.
Try It

Right triangles $ABC$ and $DEF$ are similar. Find the length in inches of side $EF$.

Since the triangles are similar, their corresponding sides are proportional. $\overline{AB}$ corresponds to $\overline{DE}$, $\overline{AC}$ corresponds to $\overline{DF}$, and $\overline{BC}$ corresponds to $\overline{EF}$. If two figures are similar, then the ratios of corresponding sides form a proportion.

$$\frac{\overline{AC}}{\overline{BC}} = \frac{x}{4}$$

Divide both sides of the equation by $3$: $x = 16$. The length of side $EF$ is 16 inches.
Scale drawings involve similar figures, so you can use proportions to solve problems which include scale drawings.

**Try It**

On a scale drawing of a doghouse, the entrance is 3.5 inches high. The scale used was 2 inches equals 1 foot. What is the height of the actual entrance to the doghouse?

Identify the quantities being compared. Let $h$ equal the height of the actual doghouse.

height in scale drawing to actual height

| _____ in. | _____ ft |
| _____ in. | $h$ ft |

Write a proportion.

\[
\frac{\text{height in scale drawing}}{\text{actual height}} = \frac{\text{_____}}{h}
\]

To find the value of $h$, solve the proportion.

First use ________________________ .

\[
\text{_____} \cdot \text{_____} = \text{_____} \cdot h
\]

Then ________________________ both sides by _____ .

$h = \text{_____}

The actual doghouse entrance is _____ feet high.

Compare 2 in. to 1 ft, and 3.5 in. to $h$ ft. The proportion is \(\frac{2}{1} = \frac{3.5}{h}\). First use cross products: \(1 \cdot 3.5 = 2h\). Then divide both sides by 2; $h = 1.75$.

The actual doghouse entrance is 1.75 feet high.

**How Can You Generate Formulas to Solve Problems?**

In math you often use formulas to solve problems. Formulas use different variables to show the relationship between quantities. Often you will know the value of all but one of the variables.

Write a formula that can be used to find the radius when given the circumference of a circle.

The formula for the circumference of a circle in the Mathematics Chart is $C = 2\pi r$. The formula is useful in this form if you know the radius of the circle and want to find the circumference.
However, if you know the circumference and want to find the radius, you can rewrite the formula.

Write the formula so that the variable representing the radius, \( r \), is on one side of the equal sign, and everything else is on the other side of the equal sign.

- Start with the formula for the circumference of a circle.
  \[ C = 2\pi r \]
- Divide both sides of the equation by \( 2\pi \).
  \[ \frac{C}{2\pi} = \frac{2\pi r}{2\pi} \]
  \[ \frac{C}{2\pi} = r \]

The formula \( \frac{C}{2\pi} = r \) can be used to find the radius when given the circumference of a circle.

---

**Try It**

Write a formula that could be used to find the side of a square, \( s \), given the perimeter of the square, \( P \).

The formula in the Mathematics Chart that relates the side of a square to its perimeter is \( P = \square \).

Rewrite the formula so that it gives \( s \) in terms of \( P \).

To do so, divide both sides of the equation by \( \square \).

\[ \frac{P}{\square} = \frac{4s}{\square} \]

\[ \frac{P}{\square} = \square \]

The new formula is \( s = \square \).

The formula is \( P = 4s \). Rewrite the formula so that it gives \( s \) in terms of \( P \).

To do so, divide both sides of the equation by \( 4: \frac{P}{4} = \frac{4s}{4} \), and \( \frac{P}{4} = s \). The new formula is \( s = \frac{P}{4} \).
How Can You Use Tables and Graphs to Interpret Formulas?

A formula like $P = 4s$ expresses a relationship between a pair of values. For every value of $s$, there will be a corresponding value of $P$. Tables and graphs can also express a relationship between pairs of values. You can use information given in a table or graph to help you see the relationship between the quantities in a formula more clearly.

The formula $\frac{f}{3} = y$ gives $y$, the number of yards in $f$ feet. The formula means that the number of feet divided by 3 equals the number of yards. You can use this formula to build a table. For each value of $f$, there is a corresponding value of $y$. For example, 3 feet = 1 yard, and 6 feet = 2 yards.

The table below shows the relationship between feet and yards.

\[
\begin{array}{|c|c|}
\hline
\text{Feet} & \text{Yards} \\
\hline
3 & 1 \\
6 & 2 \\
9 & 3 \\
15 & 5 \\
21 & 7 \\
\hline
\end{array}
\]

By considering the number pairs in the table as ordered pairs, you can also graph this relationship on a coordinate grid.

Graph the ordered pairs $(3, 1), (6, 2), (9, 3), (15, 5)$, and $(21, 7)$.

The graph shows the same relationship between feet and yards that the table shows.
Try It

This graph shows the relationship between two customary units of capacity, cups and ounces.

Build a table that represents the data in the above graph. Write the coordinates of the four points plotted on the graph.

(____, ____), (____, ____), (____, ____), (____, ____)

The x-coordinates of the points represent ______________.

The y-coordinates of the points represent ______________.

Fill in the table so that it shows the same relationship between cups and ounces that the graph shows.

<table>
<thead>
<tr>
<th>Ounces</th>
<th>Cups</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The coordinates of the four points are (8, 1), (16, 2), (24, 3), and (40, 5). The x-coordinates represent ounces. The y-coordinates represent cups.
What Is a Sequence?

A sequence of numbers is a set of numbers written in a particular order. For example, 5, 9, 13, 17 is a sequence of four numbers. The number 5 is the first term in the sequence, 9 is the second term, 13 is the third term, and 17 is the fourth term.

Here is another sequence of numbers: 3, 6, 9, 12, …

The table below shows the same sequence.

<table>
<thead>
<tr>
<th>Position</th>
<th>Value of Term</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
<tr>
<td>n</td>
<td></td>
</tr>
</tbody>
</table>

- The position column indicates a value's position in the sequence: first, second, and so on.
- The value-of-term column shows the actual numbers in the sequence: 3, 6, 9, 12, and so on.

Find a rule that tells the value for any term in the sequence using the position number.

Look for a pattern. The value of the first term is 3. The value of the second term is 6. Each value in the sequence is 3 times its position number.

What is the value of the seventh term in this sequence?
The value is 21 because $3 \cdot 7 = 21$.

What is the value of the tenth term in this sequence?
The value is 30 because $3 \cdot 10 = 30$.

What is the value of the $n$th term in this sequence?
The value is $3n$ because $3 \cdot n = 3n$.

The rule for this sequence can be represented by the expression $3n$. You can choose any position and use the rule to find the value of the number in the sequence for that position.
Look at this sequence of numbers: 5, 7, 9, 11, 13, …
Find an expression that shows the relationship between the value of any term and \( n \), its position in the sequence.

Look at this sequence in terms of the position number of each term. Is the rule \( 5n \)?

<table>
<thead>
<tr>
<th>Position</th>
<th>5n</th>
<th>Value of Term</th>
<th>Correct?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5(1) = 5</td>
<td>5</td>
<td>Yes</td>
</tr>
<tr>
<td>2</td>
<td>5(2) ≠ 7</td>
<td>10</td>
<td>No</td>
</tr>
<tr>
<td>3</td>
<td>5(3) ≠ 9</td>
<td>15</td>
<td>No</td>
</tr>
<tr>
<td>4</td>
<td>5(4) ≠ 11</td>
<td>20</td>
<td>No</td>
</tr>
<tr>
<td>5</td>
<td>5(5) ≠ 13</td>
<td>25</td>
<td>No</td>
</tr>
</tbody>
</table>

The rule \( 5n \) is not the rule for the \( n \)th term in this sequence, because the rule does not work for all terms in the sequence.

Consider the following possibility: the \( n \)th term = \( 2n + 3 \). Look at the sequence in the table below.

<table>
<thead>
<tr>
<th>Position</th>
<th>2n + 3</th>
<th>Value of Term</th>
<th>Correct?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2(1) + 3 = 5</td>
<td>5</td>
<td>Yes</td>
</tr>
<tr>
<td>2</td>
<td>2(2) + 3 = 7</td>
<td>7</td>
<td>Yes</td>
</tr>
<tr>
<td>3</td>
<td>2(3) + 3 = 9</td>
<td>9</td>
<td>Yes</td>
</tr>
<tr>
<td>4</td>
<td>2(4) + 3 = 11</td>
<td>11</td>
<td>Yes</td>
</tr>
<tr>
<td>5</td>
<td>2(5) + 3 = 13</td>
<td>13</td>
<td>Yes</td>
</tr>
</tbody>
</table>

The expression that shows the relationship between any term and \( n \), its position in this sequence, is \( 2n + 3 \).
**Try It**

Look at this sequence of numbers: 4, 6, 8, 10, 12, …

Which of these three expressions can you use as the rule to find the value of the \(n\)th term in the sequence?

\[
2n \quad n + 2 \quad 2n + 2
\]

Check the first expression, \(2n\).

When \(n = 1\), the expression \(2n = \) _______.

But the first term should be _______.

Check the second expression, \(n + 2\).

When \(n = 1\), the expression \(n + 2 = \) _______ + _______ = _______.

But the first term should be _______.

Check the third expression, \(2n + 2\).

When \(n = 1\), the expression \(2n + 2 = \) _______ + _______ = _______.

When \(n = 2\), the expression \(2n + 2 = \) _______ + _______ = _______.

When \(n = 3\), the expression \(2n + 2 = \) _______ + _______ = _______.

When \(n = 4\), the expression \(2n + 2 = \) _______ + _______ = _______.

When \(n = 5\), the expression \(2n + 2 = \) _______ + _______ = _______.

The expression _______________ can be used as the rule to find the value of the \(n\)th term in this sequence.

---

When \(n = 1\), the expression \(2n = 2\). But the first term should be 4. When \(n = 1\), the expression \(n + 2 = 1 + 2 = 3\). But the first term should be 4. When \(n = 1\), the expression \(2n + 2 = 2 + 2 = 4\). When \(n = 2\), the expression \(2n + 2 = 4 + 2 = 6\). When \(n = 3\), the expression \(2n + 2 = 6 + 2 = 8\). When \(n = 4\), the expression \(2n + 2 = 8 + 2 = 10\). When \(n = 5\), the expression \(2n + 2 = 10 + 2 = 12\). The expression \(2n + 2\) can be used as the rule to find the value of the \(n\)th term in this sequence.
**How Do You Solve Equations?**

Some equations use a variable to represent a quantity. To solve an equation containing a variable, find the value of the variable. The value of the variable is the number that makes the equation true.

In the equation $2b - 5 = 11$, the value for $b$ that makes the equation true is 8.

\[
2 \cdot 8 - 5 = 11 \\
16 - 5 = 11 \\
11 = 11
\]

The equation is true. The value 8 is the solution for $b$ in the equation $2b - 5 = 11$.

There are many different methods to find solutions for different types of equations. One way to solve equations is to use models to represent quantities. This will help you find the value of the variable that makes the equation true.

The model below represents the equation $x + 7 = 10$.

Use the model to find the value of $x$. Remove equivalent values from each side of the balance until $x$ is left by itself on one side of the scale.

The model shows that if $x + 7 = 10$, then $x = 3$. 

The model below represents the equation $8x + 5 = 3x + 15$.

Use the model to find the value of $x$ that makes the equation true.
- Remove like quantities from each side.
  Subtract (cross off) 5 circles from both sides of the equation.
  Subtract (cross off) 3 triangles from both sides of the equation.

- There are 5 triangles remaining on the left side and 10 circles remaining on the right side. Divide both sides of the equation by 5.
  - If 1 triangle = 2 circles, then $x = 2$.
  - Check to see if 2 is the solution. Replace $x$ in the equation with 2 and see if the resulting equation is true.

\[
\begin{align*}
8x + 5 &= 3x + 15 \\
8(2) + 5 &\neq 3(2) + 15 \\
16 + 5 &\neq 6 + 15 \\
21 &\neq 21
\end{align*}
\]

The equation is true. The solution is $x = 2$. 

---

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**Try It**

The model below represents the equation $4x - 3 = 2x - 9$.

Use the model to find the value of $x$.

Cross out ______ squares from each side of the equation.

Cross out ______ circles from each side of the equation.

This leaves ______ squares on the left side and ______ circles on the right side.

Since ______ squares equal ______ circles, ______ square must equal ______ circles.

The value of each circle is $-1$, so $3$ times $-1$ equals $-3$.

So the value of $x$ is ______.

Cross out $2$ squares from each side of the equation. Cross out $3$ circles from each side of the equation. This leaves $2$ squares on the left side and $6$ circles on the right side. Since $2$ squares equal $6$ circles, $1$ square must equal $3$ circles. So the value of $x$ is $-3$. 

---

Subtracting a number is the same as adding the negative of that number. For example, $x - 1 = x + (-1)$. 
How Do You Match a Problem Situation with an Equation?

Sometimes you need to determine whether a given equation represents a particular problem situation. To do this, you might have to work backwards by writing an equation for a situation. Then see if your equation matches the given equation. Or you might look at the operations in the given equation to see if they make sense based on what is explained in the problem situation.

Look at the equation $8x = 104$. Which problem situation matches this equation?

**Situation A**
Mr. Jones has a small rectangular brick patio.
One dimension is 8 feet. The perimeter of the patio is 104 feet. What is the other dimension?

Does the given equation represent Situation A?

- The formula for finding the perimeter of a rectangle is $P = 2(l + w)$.
- In this problem the perimeter is 104 ft, and one of the dimensions of the rectangle is 8 ft. Let $x$ equal the other dimension of the rectangle.
- Substitute these values in the formula $P = 2(l + w)$ to find the equation for this problem situation.

$$104 = 2(8 + x)$$

The given equation, $8x = 104$, does not represent Situation A.

**Situation B**
Mr. Jones has a small rectangular brick patio.
One dimension is 8 feet. The area of the patio is 104 square feet. What is the other dimension?

Does the given equation represent Situation B?

- The formula for finding the area of a rectangle is $A = lw$.
- In this problem the area is 104 ft$^2$, and one of the dimensions of the rectangle is 8 ft. Let $x$ equal the other dimension of the rectangle.
- Substitute these values in the formula $A = lw$ to find the equation for this problem situation.

$$104 = 8x$$

The given equation, $8x = 104$, does represent Situation B.
Try It
Frank works the same number of hours each workday. He works 5 days per week and earns $10.50 per hour. Frank earns $115.50 each week. How many hours per day does Frank work?

Does the equation \(115.50 = (5 + x) \cdot (10.50)\) match the problem situation?

First write an equation that could be used to find the amount of money Frank earns in a week.

Weekly money earned =
(Total ______ worked) \cdot (Hourly ________)

Next find what those quantities are in this problem.

The weekly money earned is $____________.

Let \(x\) equal the number of hours Frank works in 1 day.

Let _______ x equal the total number of hours he works each week.

Frank’s pay rate is $____________ per hour.

Finally, substitute these values in the formula to find the equation for this problem situation.

Weekly money earned = (Total hours worked) \cdot (Hourly pay rate)

____________ = _______ \cdot __________

Compare this equation to the given equation.

The given equation, __________ = __________ \cdot __________,
__________ match the problem situation.
How Do You Write an Equation to Describe a Problem Situation?

Sometimes in order to solve a problem you will need to write an equation that describes the problem situation. This equation must correctly represent the relationships between the various quantities described in the statement of the problem by taking the information given in everyday language and rewriting it using mathematical symbols.

Follow these steps to help create equations to describe and solve problems.

- Read the whole problem carefully.
- Identify the quantities or numbers in the problem.
- Determine which quantities you know and which you have to find.
- Express the different quantities you need to find with different letters.
- Look for relationships between the quantities described in the problem.
- Write an equation that best represents these relationships using letters, numbers, and mathematical symbols that represent mathematical operations.

Carolyn bought presents for her brothers and 2 sisters. She spent $9.50 for each brother’s present, $11.25 for each sister’s present for a total of $51.00 for all the presents. Write an equation that represents this situation and may be used to find out how many brothers Carolyn has.

- We know that $9.50 was spent on the present for each brother and $11.25 was spent on the present for each sister. We also know that the total spent for all of the presents was $51.00.
- What we do not know, and may need to find, is how many brothers Carolyn has.
- We may use \(b\) to represent the number of brothers Carolyn has.
- The amount Carolyn spent on presents for her 2 sisters equals the number of sisters Carolyn has, multiplied by how much she spent on each present: \(2(11.25)\), in dollars. The amount Carolyn spent on presents for her brothers equals the number of brothers Carolyn has, multiplied by how much she spent on each present: \(b(9.50)\), in dollars. The sum of these two amounts is 51.00, in dollars.
- The equation which represents this situation may be written as \(2(11.25) + b(9.50) = 51.00\).

Now practice what you’ve learned.
**Question 13**
A family went to a restaurant. The total cost of the meal was $48.30. The family had a coupon for 20% off the total. What was the final cost of the meal after the coupon was used?

A $9.66  
B $28.30  
C $48.10  
D $38.64

**Question 14**
Perry took a survey of 125 students at his school to see how many have their own e-mail account. Of the students surveyed, 35 students have their own e-mail account. What percent of the students surveyed have their own e-mail account?

A 90%  
B 28%  
C 72%  
D 3.57%

**Question 15**
A 2-pint bottle of soy sauce costs $1.79. If the unit price remains the same, how much will 2 gallons of soy sauce cost?

A $7.16  
B $14.32  
C $28.64  
D $57.28

**Question 16**
A designer drew a scale model of a ramp that will be used to load crates onto a truck.

What will be the length, x, of the ramp?

A $10\frac{2}{3}$ feet  
B $5\frac{1}{3}$ feet  
C $2\frac{2}{3}$ feet  
D $21\frac{1}{3}$ feet
Question 17
Triangle $XYZ$ is similar to triangle $PQR$.

Which equation could be used to find the length of side $QR$?

A $\frac{12.5}{14} = \frac{9}{x}$

B $\frac{5}{12.5} = \frac{x}{9}$

C $\frac{14}{9} = \frac{12.5}{x}$

D $\frac{5}{12.5} = \frac{9}{x}$

Question 18
Keola has a circular fish pond in his backyard that has a circumference of 36 ft. Which equation below can Keola use to calculate the radius of his fish pond?

A $r = \pi (6^2)$

B $r = 2\pi (36)$

C $r = \frac{36}{\pi}$

D $r = \frac{18}{\pi}$
Question 19

Which graph best represents the number of quarts, $x$, for different numbers of pints, $y$?

A  

B  

C  

D
Yolanda was packaging seeds for a seed company. She put the same number of seeds in each packet. She graphed the total number of seeds for different numbers of packets. Which table best represents the data in the graph?

A

<table>
<thead>
<tr>
<th>Number of Packets</th>
<th>Number of Seeds</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>30</td>
</tr>
<tr>
<td>3</td>
<td>45</td>
</tr>
<tr>
<td>5</td>
<td>75</td>
</tr>
<tr>
<td>6</td>
<td>90</td>
</tr>
<tr>
<td>8</td>
<td>120</td>
</tr>
</tbody>
</table>

B

<table>
<thead>
<tr>
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<th>Number of Seeds</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4.5</td>
</tr>
<tr>
<td>3</td>
<td>4.5</td>
</tr>
<tr>
<td>5</td>
<td>7.5</td>
</tr>
<tr>
<td>6</td>
<td>9</td>
</tr>
<tr>
<td>8</td>
<td>12</td>
</tr>
</tbody>
</table>

C

<table>
<thead>
<tr>
<th>Number of Packets</th>
<th>Number of Seeds</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
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<tr>
<td>45</td>
<td>3</td>
</tr>
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<td>75</td>
<td>5</td>
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<tr>
<td>90</td>
<td>6</td>
</tr>
<tr>
<td>120</td>
<td>8</td>
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</tbody>
</table>

D

<table>
<thead>
<tr>
<th>Number of Packets</th>
<th>Number of Seeds</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>8</td>
<td>8</td>
</tr>
</tbody>
</table>

Answer Key: page 148
Question 21
Which sequence follows the rule $3(n + 1)$, where $n$ is the number’s position in the sequence?

A 3, 6, 9, 12, 15, ...
B 4, 7, 10, 13, 16, ...
C 6, 9, 12, 15, 18, ...
D 6, 9, 12, 21, 24, ...

Question 22
In the equation $7x + 3 = 4x + 15$ modeled below, find the value of $x$ that makes the equation true.

A 2
B 3
C 4
D 5

Question 23
Which problem situation matches the equation shown below?

$\frac{128 + 145 + 139 + 157 + x}{5} = 148$

A The weights of four brothers are 128 pounds, 145 pounds, 139 pounds, and 157 pounds. Find $x$, the average weight of the brothers.
B The number of miles four people drove on trips were 128, 145, 139, and 157. Find $x$, the difference between the total sum of the distances and the number of miles for the fifth person, 148.
C The number of students in fourth through seventh grade are 128, 145, 139, and 157. Find $x$, the average number of students in each grade.
D A person bowled four games and scored 128, 145, 139, and 157. Find $x$, the score the person would need to have in the fifth game to average 148 for all five games.
Objective 3

The student will demonstrate an understanding of geometry and spatial reasoning.

For this objective you should be able to

- compare and classify two- and three-dimensional figures using geometric vocabulary and properties;
- use coordinate geometry to describe locations on a plane; and
- use geometry to model and describe the physical world.

What Are Complementary and Supplementary Angles?

Two angles are complementary if the sum of their measures is $90^\circ$.

\[ \angle NRS \text{ and } \angle SRP \text{ are complementary angles.} \]

\[ \angle A \text{ and } \angle B \text{ are also complementary angles.} \]

Two angles are supplementary if the sum of their measures is $180^\circ$.

\[ \angle TUX \text{ and } \angle XUV \text{ are supplementary angles.} \]

\[ \angle Y \text{ and } \angle Z \text{ are also supplementary angles.} \]
\( \angle ABC \) is a straight angle. Identify two angles that are complementary and two different pairs of angles that are supplementary.

- \( \angle DBC \) measures 37\(^\circ\), and \( \angle FBA \) measures 53\(^\circ\).
  
  \[
  m\angle DBC + m\angle FBA = 37^\circ + 53^\circ = 90^\circ
  \]
  
  Therefore, \( \angle DBC \) and \( \angle FBA \) are complementary angles.

- The sum of the measures of \( \angle CBD \) and \( \angle DBA \) equals the measure of \( \angle ABC \).
  
  Because \( \angle ABC \) is a straight angle, \( m\angle ABC = 180^\circ \).
  
  \[
  m\angle CBD + m\angle DBA = 180^\circ
  \]
  
  Therefore, \( \angle CBD \) and \( \angle DBA \) are supplementary angles.

- The sum of the measures of \( \angle CBF \) and \( \angle FBA \) equals the measure of \( \angle ABC \).
  
  Because \( \angle ABC \) is a straight angle, \( m\angle ABC = 180^\circ \).
  
  \[
  m\angle CBF + m\angle FBA = 180^\circ
  \]
  
  Therefore, \( \angle CBF \) and \( \angle FBA \) are supplementary angles.

---

**Try It**

If \( m\angle WXY = 72.5^\circ \), what is the measure of its supplement?

The measure of \( \angle WXY \) is ________°.

\( \angle \)______ is the supplement of \( \angle WXY \).

\[
 m\angle \text{______} + m\angle WXY = \text{______}^\circ; \ m\angle \text{______} = \text{______}^\circ
\]

The measure of the supplement of \( \angle WXY \) is ________°.

The measure of \( \angle WXY \) is 72.5°. \( \angle ZXW \) is the supplement of \( \angle WXY \).

\[
 m\angle ZXW + m\angle WXY = 180^\circ; \ m\angle ZXW = 107.5^\circ.
\]

The measure of the supplement of \( \angle WXY \) is 107.5°.
### How Can You Classify Two-Dimensional Figures?

You can classify two-dimensional figures based on information about their sides and angles. Two-dimensional figures include triangles, quadrilaterals, other polygons, and circles.

A **triangle** is a 3-sided polygon. The sum of the measures of the three angles of any triangle is $180^\circ$. Here are some types of triangles with which you should be familiar.

#### Classifying Triangles by Sides

<table>
<thead>
<tr>
<th>Type</th>
<th>Example</th>
<th>Properties</th>
</tr>
</thead>
</table>
| Scalene triangle | ![Scalene Triangle](image) | • No sides are congruent.  
                   • No angles are congruent. |
| Isosceles triangle | ![Isosceles Triangle](image) | • At least two sides are congruent.  
                   • Two angles, called the base angles, are congruent. |
| Equilateral triangle | ![Equilateral Triangle](image) | • All three sides are congruent.  
                   • All three angles are congruent. |

#### Classifying Triangles by Angles

<table>
<thead>
<tr>
<th>Type</th>
<th>Example</th>
<th>Properties</th>
</tr>
</thead>
</table>
| Right triangle | ![Right Triangle](image) | • Exactly one angle is a right angle.  
                   • The acute angles are complementary.  
                   • The side opposite the right angle, called the hypotenuse, is the longest side. |
| Acute triangle | ![Acute Triangle](image) | • All three angles are acute. |
| Obtuse triangle | ![Obtuse Triangle](image) | • Exactly one angle is an obtuse angle. |
A **quadrilateral** is a 4-sided polygon. The sum of the measures of the four angles of any quadrilateral is $360^\circ$. Some types of quadrilaterals with which you should be familiar are shown below.

### Quadrilaterals

<table>
<thead>
<tr>
<th>Type</th>
<th>Example</th>
<th>Properties</th>
</tr>
</thead>
</table>
| Parallelogram | ![Parallelogram Diagram](image) | - Both pairs of opposite sides are parallel.  
- Both pairs of opposite sides are congruent.  
- Both pairs of opposite angles are congruent.  
- Consecutive angles are supplementary. |
| Rectangle  | ![Rectangle Diagram](image) | - Both pairs of opposite sides are parallel.  
- Both pairs of opposite sides are congruent.  
- All pairs of adjacent sides are perpendicular.  
- All angles are right angles. |
| Rhombus    | ![Rhombus Diagram](image) | - Both pairs of opposite sides are parallel.  
- All sides are congruent.  
- Both pairs of opposite angles are congruent.  
- Consecutive angles are supplementary. |
| Square     | ![Square Diagram](image) | - Both pairs of opposite sides are parallel.  
- All sides are congruent.  
- All pairs of adjacent sides are perpendicular.  
- All angles are right angles. |
| Trapezoid  | ![Trapezoid Diagram](image) | - Exactly one pair of opposite sides is parallel.  
- Exactly two pairs of consecutive angles are supplementary. |
Try It

Look at these two-dimensional figures.

Which figure does not appear to have both pairs of opposite sides parallel, nor all pairs of consecutive angles supplementary?

Figure 1 appears to be a ____________________________ .
In a ____________________________ both pairs of opposite sides are _____________ , and all pairs of consecutive angles are ________________ .

Figure 2 appears to be a ____________________________ .
In a ____________________________ both pairs of opposite sides are _____________ , and all pairs of consecutive angles are ________________ .

Figure 3 appears to be a ____________________________ .
In a ____________________________ both pairs of opposite sides are _____________ , and all pairs of consecutive angles are ________________ .

Figure 4 appears to be a ____________________________ .
In a ____________________________ only _______ pair of opposite sides is parallel, and only two pairs of consecutive angles are ________________ .

Only the sides and angles in Figure _______ meet the requirements.
A face is a flat surface in the shape of a polygon. An edge is a line segment where two faces meet. A vertex is a point where three or more edges meet. The plural of vertex is vertices.

How Can You Classify Three-Dimensional Figures?

You can classify three-dimensional figures based on information about their faces, bases, edges, and vertices. Three-dimensional figures include prisms and pyramids, as well as figures with curved surfaces.

A prism is a three-dimensional figure with two parallel, congruent bases. The bases, which are also two of the faces, can be any polygon. The other faces are rectangles. A prism is named according to the shape of its bases. For example, the bases in the prism below are hexagons, so the figure is a hexagonal prism.

A pyramid is a three-dimensional figure with only one base. The base can be any polygon. The other faces are triangles. A pyramid is named according to the shape of its base. For example, the base in the pyramid below is a square, so the figure is a square pyramid.
Here are some three-dimensional figures with which you should be familiar.

### Prisms and Pyramids

<table>
<thead>
<tr>
<th>Type</th>
<th>Example</th>
<th>Properties</th>
</tr>
</thead>
</table>
| Triangular prism   | ![Triangular prism](image) | ● 5 faces  
  2 triangular bases  
  3 rectangular faces  
● 9 edges  
● 6 vertices |
| Rectangular prism  | ![Rectangular prism](image) | ● 6 faces  
  2 rectangular bases  
  4 rectangular faces  
● 12 edges  
● 8 vertices |
| Cube               | ![Cube](image) | ● 6 faces  
  2 square bases  
  4 square faces  
● 12 edges  
● 8 vertices |
| Square pyramid     | ![Square pyramid](image) | ● 5 faces  
  1 square base  
  4 triangular faces  
● 8 edges  
● 5 vertices |
| Triangular pyramid | ![Triangular pyramid](image) | ● 4 faces  
  1 triangular base  
  3 triangular faces  
● 6 edges  
● 4 vertices |
You should also be familiar with three-dimensional figures that have curved surfaces. These figures include cylinders, cones, and spheres. You can classify these three-dimensional figures based on information about their bases and surfaces.

Three-Dimensional Figures with Curved Surfaces

<table>
<thead>
<tr>
<th>Type</th>
<th>Example</th>
<th>Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cylinder</td>
<td><img src="image" alt="Cylinder" /></td>
<td>● 2 circular bases&lt;br&gt;● 1 curved surface</td>
</tr>
<tr>
<td>Cone</td>
<td><img src="image" alt="Cone" /></td>
<td>● 1 circular base&lt;br&gt;● 1 curved surface&lt;br&gt;● 1 vertex</td>
</tr>
<tr>
<td>Sphere</td>
<td><img src="image" alt="Sphere" /></td>
<td>● 1 curved surface</td>
</tr>
</tbody>
</table>

Try It

Look at the figure below.

The figure shown has ______ bases, each shaped like a ____________.

The 2 bases of the figure are parallel and ____________.

The figure has ______ rectangular faces.

The figure has ______ edges.

The figure has ______ vertices.

The figure is a ____________________________.

The figure shown has 2 bases, each shaped like a pentagon. The 2 bases of the figure are parallel and congruent. The figure has 5 rectangular faces. The figure has 15 edges. The figure has 10 vertices. The figure is a pentagonal prism.
What Are Similar Figures?

Two geometric figures are similar if the following conditions are true:

- Both figures are the same shape.
- The corresponding angles of the figures are congruent.
- The ratios of the lengths of corresponding sides are equal, so they form a proportion.

Rectangle $EFGH$ is similar to rectangle $WXYZ$.

$\angle E$ corresponds to $\angle W$.
$\angle F$ corresponds to $\angle X$.
$\angle G$ corresponds to $\angle Y$.
$\angle H$ corresponds to $\angle Z$.

Side $EF$ corresponds to side $WX$.
Side $FG$ corresponds to side $XY$.
Side $GH$ corresponds to side $YZ$.
Side $HE$ corresponds to side $ZW$.

All the angles are right angles, so all corresponding angles are congruent.

- The length of the larger rectangle is 9 units, and the length of the smaller rectangle is 3 units. The ratio is $\frac{9}{3}$.
- The width of the larger rectangle is 6 units, and the width of the smaller rectangle is 2 units. The ratio is $\frac{6}{2}$.

The proportion $\frac{9}{3} = \frac{6}{2}$ is true, so the ratios of the lengths of the corresponding sides are proportional.
Parallelogram $ABCD$ is similar to parallelogram $KLMN$. Write and solve a proportion to find $x$, the length of side $AD$.

- Corresponding sides of similar figures are proportional.
  - $\overline{AB}$ corresponds to $\overline{KL}$.
  - $\overline{AD}$ corresponds to $\overline{KN}$.
- Write a proportion that compares the ratios of corresponding sides.
  \[
  \frac{\text{smaller}}{\text{larger}} = \frac{AB}{KL} = \frac{AD}{KN}
  \]
- Substitute the values into the proportion.
  \[
  \frac{2.5}{5} = \frac{x}{2}
  \]
- Use cross products.
  \[
  2.5 \cdot 2 = 5x
  \]
  \[
  5 = 5x
  \]
- Divide both sides by 5 to find the value of $x$.
  \[
  \frac{5}{5} = \frac{5x}{5}
  \]
  \[
  1 = x
  \]

The length of side $AD$ is 1 unit.
Right triangles $XYZ$ and $RST$ are similar.

Find $n$, the length of side $RT$.

- Side $XY$ corresponds to side $RS$.
- Side $XZ$ corresponds to side $RT$.
- Write a proportion that compares the ratios of corresponding sides.

\[ \frac{XY}{RS} = \frac{XZ}{RT} \]

- Substitute the values into the proportion.

\[ \frac{2}{5} = \frac{8}{n} \]

- Solve the proportion.

\[ 2n = 5 \cdot 8 \quad \text{Use cross products to find the value of } n. \]

\[ 2n = 40 \]
\[ \frac{2n}{2} = \frac{40}{2} \]
\[ n = 20 \]

The length of side $RT$ is 20 units.

**Try It**

Which rectangle is similar to rectangle $FGHI$?

- Corresponding angles of similar figures are congruent.
- All the angles in a rectangle are ___________ angles, so both of the other rectangles have angles that are _________________ to the corresponding angles in rectangle $FGHI$. 
The ratios of the lengths of corresponding sides of similar figures form proportions. Match up the corresponding sides of rectangles $FGHI$ and $NOPR$.

Side $FG$ corresponds to side $NO$.

Side $FI$ corresponds to side _______.

Express this as two ratios.

\[
\frac{FG}{NO} = \frac{10}{\text{______}} \quad \text{and} \quad \frac{FI}{NR} = \frac{\text{______}}{5}
\]

Write a proportion that compares the ratios of the corresponding sides. If the proportion is true, the cross products will be equal.

\[
\frac{10}{\text{______}} = \frac{\text{______}}{5}
\]

\[
56 \neq \text{______}
\]

The proportion is not true. Rectangles $FGHI$ and $NOPR$ are _______ similar.

Match up the corresponding sides of rectangles $FGHI$ and $JKLM$.

Side $FG$ corresponds to side $JK$.

Side $FI$ corresponds to side _______.

Express this as two ratios.

\[
\frac{FG}{JK} = \frac{\text{______}}{\text{______}} \quad \text{and} \quad \frac{FI}{JM} = \frac{\text{______}}{\text{______}}
\]

Write a proportion that compares the ratios of the corresponding sides. If the proportion is true, the cross products will be equal.

\[
\frac{10}{5} = \frac{7}{3.5}
\]

\[
35 = \text{______}
\]

The proportion is true. Rectangles $FGHI$ and $JKLM$ are _______.

All the angles in a rectangle are right angles, so both of the other rectangles have angles that are congruent to the corresponding angles in rectangle $FGHI$.

Side $FI$ corresponds to side $NR$.

\[
\frac{FG}{NO} = \frac{10}{8}; \quad \frac{FI}{NR} = \frac{7}{\text{______}}; \quad \frac{10}{8} = \frac{7}{\text{______}}; \quad 56 \neq 50.
\]

Rectangles $FGHI$ and $NOPR$ are not similar. Side $FI$ corresponds to side $JM$.

\[
\frac{FG}{JK} = \frac{10}{5}; \quad \frac{FI}{JM} = \frac{7}{3.5}; \quad \frac{10}{5} = \frac{7}{3.5}; \quad 35 = 35.
\]

Rectangles $FGHI$ and $JKLM$ are similar.
How Can You Locate and Name Points on a Coordinate Plane?

A coordinate grid is used to locate and name points on a plane. The coordinate grid is formed by two perpendicular number lines. A point is located by using an **ordered pair** of numbers. The two numbers that form the ordered pair are called the **coordinates** of the point.

The **x-axis** and **y-axis** divide the coordinate plane into 4 regions called **quadrants**. The quadrants are usually referred to by the Roman numerals I, II, III, and IV.
Quadrilateral $ABCD$ is drawn on the coordinate grid below.

What are the coordinates of points $A$, $B$, $C$, and $D$?

- Point $A$ is located at $(5, 6)$. It is 5 units to the right of the origin and 6 units above the origin.
- Point $B$ is located at $(3, -7)$. It is 3 units to the right of the origin and 7 units below the origin.
- Point $C$ is located at $(-8, -2)$. It is 8 units to the left of the origin and 2 units below the origin.
- Point $D$ is located at $(-6, 4)$. It is 6 units to the left of the origin and 4 units above the origin.
What are the coordinates of the point that is 4 units to the left of and 5 units below point N?

- The coordinates of point N are (3, 5) because point N is 3 units to the right of the origin and 5 units above the origin.
- The new point is 4 units to the left of point N. If you move 4 units to the left of 3, you are at −1. The x-coordinate is −1.
- The new point is 5 units below point N. If you move 5 units down from 5, you are at 0. The y-coordinate is 0.

The coordinates of the point 4 units to the left of and 5 units below point N are (−1, 0).
Try It

Look at the points plotted on the coordinate plane. Which point has the coordinates $(−8, −4)$?

Point $P$ is ______ units to the right of the origin.
Point $P$ is ______ unit below the origin.
The coordinates of point $P$ are (_____, ______).

Point $V$ is ______ units to the right of the origin.
Point $V$ is ______ units above the origin.
The coordinates of point $V$ are (_____, ______).

Point $R$ is ______ unit to the left of the origin.
Point $R$ is ______ units above the origin.
The coordinates of point $R$ are (_____, ______).

Point $S$ is ______ units to the left of the origin.
Point $S$ is ______ units above the origin.
The coordinates of point $S$ are (_____, ______).

Are the numbers below the origin positive or negative?
Point $T$ is ______ units to the left of the origin.

Point $T$ is ______ units below the origin.

The coordinates of point $T$ are (_____, _____).

Only point ______ has the coordinates $(-8, -4)$.

Point $P$ is 4 units to the right of the origin. Point $P$ is 1 unit below the origin. The coordinates of point $P$ are $(4, -1)$.

Point $V$ is 0 units to the right of the origin. Point $V$ is 4 units above the origin. The coordinates of point $V$ are $(0, 4)$.

Point $R$ is 1 unit to the left of the origin. Point $R$ is 3 units above the origin. The coordinates of point $R$ are $(-1, 3)$.

Point $S$ is 6 units to the left of the origin. Point $S$ is 0 units above the origin. The coordinates of point $S$ are $(-6, 0)$.

Point $T$ is 8 units to the left of the origin. Point $T$ is 4 units below the origin. The coordinates of point $T$ are $(-8, -4)$.

Only point $T$ has the coordinates $(-8, -4)$. 
**What Is a Translation?**

A **translation** is a movement of a figure or point along a line. A translation can be described by how many units a figure is moved to the left or right and how many units it is moved up or down. A figure and its translated image are always congruent.

If triangle $ABC$ is translated 2 units to the right and 10 units up, what will be the new coordinates of point $A$?

- The coordinates of point $A$ are $(-7, -6)$.
- The $x$-coordinate of point $A$ is $-7$. When point $A$ is translated 2 units to the right, its new $x$-coordinate will be $-5$.
- The $y$-coordinate of point $A$ is $-6$. When point $A$ is translated 10 units up, its new $y$-coordinate will be 4.

The new coordinates of point $A$ will be $(-5, 4)$. 
Try It

If quadrilateral $ABCD$ is translated 7 units to the left and 3 units down, what will be the coordinates of the vertices of the new figure?

The coordinates of point $A$ are (_____, ____).  
After translation, the new coordinates of point $A$ will be (_____, ____).

The coordinates of point $B$ are (_____, ____).  
After translation, the new coordinates of point $B$ will be (_____, ____).

The coordinates of point $C$ are (_____, ____).  
After translation, the new coordinates of point $C$ will be (_____, ____).

The coordinates of point $D$ are (_____, ____).  
After translation, the new coordinates of point $D$ will be (_____, ____).

The coordinates of point $A$ are (3, 2). After translation, the new coordinates of point $A$ will be (−4, −1). The coordinates of point $B$ are (6, 7). After translation, the new coordinates of point $B$ will be (−1, 4). The coordinates of point $C$ are (8, 4). After translation, the new coordinates of point $C$ will be (1, 1). The coordinates of point $D$ are (5, −5). After translation, the new coordinates of point $D$ will be (−2, −8).
What Is a Reflection?

A reflection of a figure is its mirror image. A figure is reflected across a line called the line of reflection. The line of reflection serves as the mirror on which the figure is reflected. A figure and its reflected image are always congruent.

Each point of the reflected image is the same distance from the line of reflection as the corresponding point of the original figure, but it is on the opposite side of the line of reflection.

If the point \((2, 3)\) is reflected across the \(y\)-axis, what will its new coordinates be?

- The \(y\)-coordinate of the point will be unchanged because the point is being reflected across the \(y\)-axis. The new point will have a \(y\)-coordinate of 3.
- The \(x\)-coordinate of the point is 2 units to the right of the \(y\)-axis, so the \(x\)-coordinate of the new point will be 2 units to the left of the \(y\)-axis. The new point will have an \(x\)-coordinate of \(-2\).

The coordinates of the new point will be \((-2, 3)\). The point \((2, 3)\) and its image \((-2, 3)\) are equally distant from the line of reflection, the \(y\)-axis.
How Can You Sketch a Three-Dimensional Figure When Given Its Top, Front, and Side Views?

Sometimes you are given the top, front, and side views of a three-dimensional figure. To sketch the figure, you need to imagine how these different views are related to the figure.

- **Top View:** What would the figure look like if you were standing over it, looking straight down on it?
- **Front View:** What would the figure look like if you were standing in front of it, looking at it straight on?
- **Side View:** What would the figure look like if you were standing at the side of it, looking at it straight on?

Look at these views of a three-dimensional figure: top view, front view, and side view.

Which solid figure do the views above represent?

To determine which three-dimensional figure is represented by the views above, compare the different views to the solids.

The top view is a triangle. Imagine yourself above each of the figures. Imagine looking straight down on them.

- Only Figures 2 and 3 have a triangle as their top view, so Figures 1 and 4 cannot be the correct answer.
- Consider the other two views for Figures 2 and 3.

The front view is a rectangle that is slightly longer than it is wide. Imagine yourself in front of each of the figures. Imagine yourself looking straight at them.

- The front view of Figure 2 is a rectangle, but its length is much greater than its width. It does not match the front view shown. Figure 2 cannot be the correct answer.
Objective 3

Try It

Look at the top, front, and side views of a three-dimensional figure.

- The front view of Figure 3 is also a rectangle. The front view of Figure 3 matches the front view given.
  
  Figure 3
  Front

- The side view of Figure 3 also matches the side view given.
  
  Figure 3
  Side

Figure 3 is the three-dimensional figure that the three views represent.

Try It

Look at the top, front, and side views of a three-dimensional figure.

Which figure below is represented by the views above?

- The top view given is a ____________________________ .
  
  Figure 1
  Top

- The front view given is a ____________________________ .
  
  Figure 2
  Front

- The side view given is a ____________________________ .
  
  Figure 3
  Side

Figure ______ is best represented by the views above.

The top view given is a rectangle. The front view given is a triangle. The side view given is a rectangle. Figure 1 is best represented by the views above.
What Is a Net?

A net is a two-dimensional pattern of a three-dimensional figure. When the net is folded up, it forms the figure. You should be able to match a drawing of a three-dimensional figure with its net.

Suppose you want to make a net of the box shown below. The box is shaped like a rectangular prism. One way to make a net of the box is shown below.

- The first step in drawing the net is to fold down the top and the back panel of the box and place them on the same plane as the bottom of the box. The dotted lines show where the box was folded.

- The next step is to fold down the front of the box. Now the top, back, bottom, and front of the box form one long strip.

- The last step is to fold down the sides of the box.

The net includes all the faces of the box. You could reconstruct the box by folding the net along the dotted lines.
How Can You Solve Problems Using Geometric Concepts and Properties?

To solve problems that involve geometric concepts, you must first understand the problem. Then identify the quantities involved and the relationships between them. Finally, solve for the missing information.

A crew of workers is pouring cement for the foundation of a road. Each slab of the foundation is a rectangular prism that measures 12.25 feet by 10 feet by 1.5 feet. If the job requires 8 slabs, how many cubic feet of cement will the crew need?

Find the volume of 1 slab.

- The cement slab is a rectangular prism. Use the formula for the volume of a rectangular prism.

\[ V = lwh \]
• Identify what the variables represent.
  \[ l = \text{length} = 12.25 \text{ ft} \]
  \[ w = \text{width} = 10 \text{ ft} \]
  \[ h = \text{height} = 1.5 \text{ ft} \]

• Substitute the values you know into the formula and simplify.
  \[ V = (12.25)(10)(1.5) \]
  \[ V = 183.75 \text{ ft}^3 \]

Each slab requires 183.75 cubic feet of cement.

Find the volume of 8 slabs. Multiply to combine equal parts.
  \[ 183.75 \cdot 8 = 1,470 \]

The crew will need 1,470 cubic feet of cement.

**Try It**

The Hernández family bought a piece of property with a pond on it. They measured the distances shown below and used the fact that \( \triangle DCE \) is similar to \( \triangle BCA \) to find the width of the pond. What is the width of the pond, \( w \), in feet?

Side \( AB \) corresponds to side ______ .

Side \( AC \) corresponds to side ______ .

Write a proportion that compares the ratios of corresponding sides.

\[ \underline{} \quad = \quad \underline{} \]
Substitute the values from the problem into the proportion.

\[
\begin{array}{c}
\frac{w}{w} = \frac{24}{w} \\
\end{array}
\]

Solve the proportion.

\[
\begin{array}{c}
\frac{6}{w} \cdot \frac{2}{24} = \frac{144}{2w} = \frac{72}{w} \\
\end{array}
\]

The pond on the Hernández property is _______ feet wide.

Side \(AB\) corresponds to side \(ED\). Side \(AC\) corresponds to side \(EC\).

\[
\begin{align*}
\frac{AB}{ED} &= \frac{AC}{EC} \\
\frac{6}{w} &= \frac{2}{24} \\
6 \cdot 24 &= 2w \\
144 &= 2w \\
72 &= w \\
\end{align*}
\]

The pond on the Hernández family's property is 72 feet wide.

**Now practice what you’ve learned.**
**Question 24**

The measure of $\angle Y$ is 15.5°. What is the measure of its complement in degrees?

Record your answer and fill in the bubbles. Be sure to use the correct place value.

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**Question 25**

Triangle $MNR$ is similar to triangle $MBA$.
Which segment corresponds to $MN$?

A $\overline{AB}$
B $\overline{BN}$
C $\overline{MB}$
D $\overline{MA}$

**Question 26**

Which of the following statements is always true about parallelograms?

A All angles are congruent.
B All sides are congruent.
C Adjacent sides are perpendicular.
D Opposite sides are parallel.

**Question 27**

Which of these describes the faces of an octagonal pyramid?

A 8 rectangular faces, 2 octagonal bases
B 8 triangular faces, 1 octagonal base
C 6 triangular faces, 1 octagonal base
D 8 rectangular faces, 1 octagonal base
Question 28
In parallelogram $DEFG$, $m\angle E = 70^\circ$. What is the measure of $\angle F$ in degrees?
Record your answer and fill in the bubbles. Be sure to use the correct place value.

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Question 29
Triangle $MNR$ is similar to triangle $MBA$. Which proportion can be used to find the value of $x$?

A $\frac{6}{3} = \frac{x}{10}$
B $\frac{4}{8} = \frac{10}{x}$
C $\frac{6}{3} = \frac{10}{x}$
D $\frac{x}{3} = \frac{6}{10}$
Question 30
What are the coordinates of the reflection of point \( R \) across the \( x \)-axis?

A \((-3, -2)\)
B \((-2, -3)\)
C \((-2, 3)\)
D \((2, -3)\)

Question 31
If the polygon in the grid below is translated to the right 1 unit and down 7 units, what will be the new coordinates of point \( F \)?

A \((0, 9)\)
B \((0, 7)\)
C \((6, 1)\)
D \((8, 1)\)
Question 32
Look at the top, front, and side views of a three-dimensional figure.

Which figure is represented by the views above?

A  
B  
C  
D

Question 33
Which three-dimensional figure is represented by the net shown below?

A  Hexagonal prism
B  Hexagonal pyramid
C  Pentagonal prism
D  Pentagonal pyramid
Objective 4

The student will demonstrate an understanding of the concepts and uses of measurement.

For this objective you should be able to solve problems involving estimation and measurement of length, area, and volume.

How Can You Use Estimation in Measurement Problems?

In some problems you may be given only approximate values for measurements. In these cases you must estimate an answer. For example, some measurement problems ask about how many or approximately how long. Use estimation when solving such problems.

You can also estimate the answer to any problem before you find the exact answer. One way to estimate is to round the numbers in a problem before working it out. The estimate tells you approximately what the answer will be. If you estimate first, you will know whether the answer you calculate is reasonable. For example, some problems ask whether a certain number is a reasonable answer to a problem. Use estimation to answer such questions.

Another type of estimation involves pi, or \( \pi \). The value of \( \pi \) is based on the ratio of the measures of the circumference and diameter of a circle, and is given in the Mathematics Chart as 3.14 or \( \frac{22}{7} \).

A circle is the set of all points in a plane that are the same distance from a given point called the center.

- A radius \( (r) \) of a circle is a line segment joining the center of the circle and any point on the circle. The radius of a circle is half the diameter, or \( r = \frac{1}{2}d \).
- A diameter \( (d) \) of a circle is a line segment that joins any two points on the circle and passes through the center of the circle. The diameter of a circle is twice the radius, or \( d = 2r \).
- The circumference \( (C) \) of a circle is the distance around the circle.

3.14 and \( \frac{22}{7} \) are only estimates of the value of \( \pi \). The actual value of \( \pi \) is a number that starts 3.1415926… and continues forever without repeating a pattern.

Whenever pi is replaced with an estimate in a formula, the answer is not exact.
Michelle is designing a circular sign. The sign will have a circumference of 48 inches. What is the minimum width to the nearest inch of the piece of wood from which she can cut the sign?

- The minimum width of the piece of wood she can use is equal to the diameter of the circle.
- Use the formula for the circumference of a circle, \( C = \pi d \), to find the diameter. The formula for the circumference of a circle contains \( \pi \), so the answer you calculate will be approximate.
- Substitute 48 for \( C \) and 3.14 for \( \pi \).

\[
48 \approx 3.14d
\]

- Solve for \( d \).

\[
\frac{48}{3.14} \approx \frac{3.14d}{3.14} \quad \text{Divide both sides by 3.14.}
\]

\[
15.29 \approx d
\]

The diameter of the circle is about 15.29 inches. Therefore, the minimum width to the nearest inch of the piece of wood from which Michelle can cut the sign is 16 inches.
Try It

John plans to lay bricks around the edge of a rectangular garden. The garden is 12 feet long and 8.5 feet wide. Each brick is 10 inches long. What is the minimum number of bricks he will need to buy to complete the project?

Use the formula for the perimeter of a rectangle to find the distance around the garden:

\[ P = 2(l + w) \]

\[ P = 2(12 + 8.5) \]

\[ P = 2(20.5) \]

\[ P = 41 \]

The perimeter of the garden is 41 feet. The length of the bricks is given in inches, but the perimeter is expressed in feet. Convert the perimeter to an equivalent number of inches:

1 foot = 12 inches

41 feet \( \times \) 12 inches per foot = 492 inches

Divide 492 by 10 to find the number of bricks he will need.

\[ \frac{492}{10} = 49.2 \]

John will need 49.2 bricks. Since John cannot buy part of a brick, he will need to buy at least 50 bricks to complete the project.
How Can You Use Formulas to Solve Problems?

The Mathematics Chart lists formulas for perimeter, circumference, area, and volume.

When using a formula to solve a problem, follow these steps:

- Identify the formula that applies to the problem you are solving.
- Identify what the variables in the formula stand for.
- Substitute the variables in the formula with their values from the problem.
- Perform the calculations. Remember to use the correct order of operations.
- State the solution to the problem using the appropriate units of measurement.

Connie wants to sew some fringe across both short ends of a rectangular rug. The perimeter of the rug is 7 yards. The length of the rug is 2 yards. How many inches of fringe will Connie need?

- Identify the formula that applies to the problem you are solving.
  
  The problem involves the perimeter of a rectangle: \( P = 2l + 2w \).

- Identify what the variables in the formula stand for.
  
  \( P \) stands for perimeter, \( l \) stands for length, and \( w \) stands for width.

- Replace the variables in the formula with their values from the problem.
  
  \[
  P = 2l + 2w \\
  7 = 2(2) + 2w
  \]

- Solve for \( w \).
  
  \[
  7 = 4 + 2w \\
  -4 = -4 \\
  3 = 2w \\
  \frac{3}{2} = \frac{2w}{2} \\
  1.5 = w
  \]

- State the solution to the problem using the appropriate units of measurement.
Pat is painting the shaded part of the larger circle below. Approximately how many square inches will he paint?

- Find the area of the shaded part of the larger circle by subtracting the area of the circle with a radius of 4 inches from the area of the circle with a radius of 10 inches.
- Use the formula for the area of a circle, \( A = \pi r^2 \).
- Find the area of the circle with a radius \( r \) of 10 inches. Use 3.14 as an estimate of the value of \( \pi \).
  \[
  A \approx (3.14)(10^2) \\
  A \approx (3.14)(100) \approx 314 \text{ in.}^2
  \]
- Find the area of the circle with a radius of 4 inches.
  \[
  A \approx (3.14)(4^2) \\
  A \approx (3.14)(16) \approx 50.24 \text{ in.}^2
  \]
- To find the area of the shaded part, subtract the area of the 4-inch circle from the area of the 10-inch circle.
  \[
  314 - 50.24 \approx 263.76 \text{ in.}^2
  \]

Pat will paint about 264 square inches.

Connie will need \( 2(1.5) = 3 \) yards of fringe. The problem asks how many inches of fringe Connie will need, so convert 3 yards to inches.

Remember that 12 inches = 1 foot, and 3 feet = 1 yard.

\[
\left( \frac{3 \text{ feet}}{1 \text{ yard}} \right) \cdot \left( \frac{12 \text{ inches}}{1 \text{ foot}} \right) = \frac{36 \text{ inches}}{1 \text{ yard}}
\]

\[
3 \text{ yards} \cdot \frac{36 \text{ inches}}{1 \text{ yard}} = 108 \text{ inches}
\]

Connie will need 108 inches of fringe.
In parallelogram $ABCD$, $BE$ is perpendicular to $AD$.

What is the area of triangle $ABD$?

- Identify the formula that applies to the problem you are solving.
  The problem involves the area of a triangle. The formula for the area of a triangle is $A = \frac{1}{2}bh$.

- Identify what the variables in the formula represent.
  $A$ stands for area, $b$ stands for base, and $h$ stands for height.

- Since $BE$ is perpendicular to $AD$, $BE$ is the height of triangle $ABD$.

- Use the ruler on the Mathematics Chart to measure the length of $BE$ and the length of $AD$ in centimeters.
  $BE = 3$ centimeters and $AD = 6$ centimeters.

- Replace the variables in the formula with their values from the problem.
  $$A = \frac{1}{2}bh$$
  $$A = \frac{1}{2} \cdot 6 \cdot 3$$

- Perform the calculations called for in the formula.
  $$A = 9 \text{ cm}^2$$

- State the solution to the problem using the appropriate units of measurement.
  The area of triangle $ABD$ is 9 square centimeters.
Kim wants to know what volume of rice she can store in a container shaped like a rectangular prism, pictured below.

What is the volume of this rectangular prism?

- Identify the formula that applies to the problem she is working.
  The problem involves the volume of a rectangular prism. The formula for the volume of a rectangular prism is $V = Bh$.

- Identify what the variables in the formula represent.
  $V$ stands for volume, $B$ stands for the area of the base, and $h$ stands for the height of the prism.

We can use the shaded region as the base of the rectangular prism. Since the base is a rectangle, we can use $B = lw$, where $l$ is the length of the rectangle and $w$ is its width.

- Replace the variables in the formula for the base with the dimensions shown in the picture.
  $B = 20 \text{ cm} \cdot 15 \text{ cm}$

- Perform the calculations called for in the formula for the base.
  $B = 300 \text{ cm}^2$

- Replace the variables with the calculated value for the base and the value for the height shown in the picture.
  $V = 300 \text{ cm}^2 \cdot 30 \text{ cm}$

- Perform the calculations called for in the volume formula.
  $V = 9,000 \text{ cm}^3$

- State the solution to the problem using the appropriate units of measurement.

The volume of the rectangular prism is 9,000 cubic centimeters.
Try It
André has two containers: Container A is a rectangular prism, and Container B is a cylinder. Container A is filled with water. André wants to pour all the water into Container B. Will Container B hold all the water from Container A?

Container A is a rectangular prism. The formula for the volume of a rectangular prism is $V = lwh$.

$V_A = 20 \cdot 20 \cdot 45 = 18,000 \text{ cm}^3$

Container B is a cylinder. The formula for the volume of a cylinder is $V = \pi r^2h$.

The radius ($r$) is equal to 20 cm. Use 3.14 as an approximate value for $\pi$.

$V_B \approx 3.14 \cdot 20^2 \cdot 24 \approx 30,144 \text{ cm}^3$

The water in Container A will fit in Container B, because 18,000 cm$^3$ is less than 30,144 cm$^3$.

Now practice what you’ve learned.
Question 34
An architect is looking at plans for a new building. One rectangular window in the building will measure 3 feet by 4.5 feet. If he replaces it with a circular window that has a diameter of 4 feet, about how much less will the area of the circular window be than the area of the rectangular window?

A 1.06 ft²  
B 13.5 ft²  
C 0.94 ft²  
D 36.74 ft²

Answer Key: page 150

Question 35
A rectangular deck has a perimeter of 50 feet. The width of the deck is 7 feet. What is the area of the deck?

A 18 ft²  
B 126 ft²  
C 50 ft²  
D 350 ft²

Answer Key: page 150

Question 36
Mr. Hall wants to cover his living room floor with new carpet. His living room floor is a rectangle measuring 22 feet by 18 feet. What is the minimum number of square yards of carpet he will need to buy?

A 9 yd²  
B 44 yd²  
C 3,564 yd²  
D 720 yd²

Answer Key: page 150

Question 37
At a packaging factory, workers pack small boxes into large crates. The boxes are cubes measuring 4 inches on each edge. It takes 81 boxes to fill each crate. What is the volume of each crate in cubic inches?

A 324 in.³  
B 1,296 in.³  
C 5,184 in.³  
D 64 in.³

Answer Key: page 150

Question 38
George has a large circular table with a circumference of 18 feet. What is the approximate area of the table?

A 108 ft²  
B 12 ft²  
C 27 ft²  
D 36 ft²

Answer Key: page 150
**Question 39**
Mr. Borders wants to make 32 hamburger patties for a barbecue. If each hamburger patty requires \( \frac{5}{4} \) ounces of ground beef, how much ground beef to the nearest pound will he need to buy?

A  10 lb  
B  68 lb  
C  168 lb  
D  11 lb

**Question 40**
A grain bag contains 128 cups of horse feed. If Victor feeds \( \frac{2}{3} \) cups of grain from the bag to each of 4 horses, how many cups of grain will be left in the bag?

A  \( \frac{118}{3} \) c  
B  \( \frac{121}{3} \) c  
C  \( \frac{119}{3} \) c  
D  \( \frac{117}{3} \) c

**Question 41**
Allan filled an aquarium shaped like a rectangular prism with water. The aquarium was 2 inches wide, 7 inches long, and \( 6 \frac{1}{2} \) inches high. What would be the approximate height of a cylinder that would hold the same volume of water if the cylinder has a radius of 4 inches?

A  1.8 in.  
B  1.0 in.  
C  3.6 in.  
D  0.8 in.
Objective 5

The student will demonstrate an understanding of probability and statistics.

For this objective you should be able to

- recognize that physical or mathematical models can be used to describe the experimental and theoretical probability of real-life events;
- understand that the way a set of data is displayed influences its interpretation; and
- use measures of central tendency and range to describe a set of data.

What Is a Sample Space?

The **sample space** for a probability experiment is the set of all possible outcomes. A sample space can be shown as a list, a table, or a diagram of all possible outcomes.

An **event** is any collection of outcomes in an experiment. A **simple event** is an event that consists of a single outcome. For example, in a probability experiment, a coin is tossed. The event is the tossing of the coin; the simple event could be that the coin landed heads up.

Some experiments involve two or more simple events. An experiment with multiple simple events is called a **composite experiment**. An example of a composite experiment would be an experiment in which a coin is tossed and then a spinner is spun. An event for this experiment might be (Heads, 2), indicating that the coin landed heads up and the spinner stopped on the number 2. When you construct a sample space for a composite experiment, each possibility for the first simple event is paired with each possibility for the second.

Mary, Jim, and Brett are playing a game that uses two spinners. The first spinner has 2 sections: “Go ahead” and “Go back.” The second spinner has 3 sections: “1 space,” “2 spaces,” and “3 spaces.” Each player spins both spinners to determine the next move. Find all the possible outcomes.
A theme park provides its workers with several choices of uniform. Shirts come in green, red, blue, and black. Shorts come in white, khaki, and gray.

Build a sample space to show all the possible outcomes. Use the lines below to show the sample space.

<table>
<thead>
<tr>
<th>Shirt, shorts</th>
<th>Shirt, shorts</th>
<th>Shirt, shorts</th>
</tr>
</thead>
<tbody>
<tr>
<td>green, white</td>
<td>green, khaki</td>
<td>green, ______</td>
</tr>
<tr>
<td>______, ______</td>
<td>______, ______</td>
<td>______, ______</td>
</tr>
<tr>
<td>______, ______</td>
<td>______, ______</td>
<td>______, ______</td>
</tr>
<tr>
<td>______, ______</td>
<td>______, ______</td>
<td>______, ______</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Shirt, shorts</th>
<th>Shirt, shorts</th>
<th>Shirt, shorts</th>
</tr>
</thead>
<tbody>
<tr>
<td>green, white</td>
<td>green, khaki</td>
<td>green, gray</td>
</tr>
<tr>
<td>red, white</td>
<td>red, khaki</td>
<td>red, gray</td>
</tr>
<tr>
<td>blue, white</td>
<td>blue, khaki</td>
<td>blue, gray</td>
</tr>
<tr>
<td>black, white</td>
<td>black, khaki</td>
<td>black, gray</td>
</tr>
</tbody>
</table>

There are a total of 6 possible outcomes for this experiment.
**How Can You Select an Appropriate Way to Represent Data?**

You can represent data in a variety of ways, including tables, line graphs, bar graphs, and circle graphs. When data are organized and displayed in a graph or diagram, it is easier to see relationships between the pieces of data. The type of graph you use depends on the data you are representing and the relationships you want to analyze.

A **line plot** represents a set of data by showing how often a piece of data appears in that set. It consists of a number line that includes the values of the data set. An X is placed above the corresponding value each time that value appears in the data set.

A student recorded the lengths (in inches) of the fish he and his family caught and released during a fishing contest.

14, 10, 16, 14, 18, 20, 14, 10, 16, 16, 10, 16, 14, 10, 16, 18, 14, 16, 18, 16

The data were then rearranged from smallest to largest value.

10, 10, 10, 10, 14, 14, 14, 14, 14, 16, 16, 16, 16, 16, 16, 16, 18, 18, 18, 20

A line plot was made from the resulting data set, as shown below. Notice that only one of the fish was 20 inches long and that there is one X above the 20 on the line plot. Similarly, four of the fish were 10 inches long, and there are 4 Xs above the 10 on the line plot.

You can use this line plot to make the following observations.

- The fish vary in length from 10 to 20 inches.
- The length recorded the least was 20 inches.
- The length recorded the most was 16 inches.
A **bar graph** uses either vertical or horizontal bars of different heights or lengths to display data. A bar graph has a scale and labels so that the reader can tell what the bars represent. Bar graphs are useful for analyzing and comparing data.

Last week Julie studied for 3 hours on Monday, 2 hours on Tuesday, $2\frac{1}{2}$ hours on Wednesday, $3\frac{1}{2}$ hours on Thursday, and 1 hour on Friday. The graph below helps you compare the number of hours she studied on each day.

- The scale on the horizontal axis shows the number of hours Julie spent studying.
- The vertical axis shows the days of the week for which there are data.
- The length of each bar shows the number of hours Julie studied on that particular day. For example, the bar for Wednesday ends halfway between 2 and 3 on the scale. This shows that Julie studied for $2\frac{1}{2}$ hours on that day.
- When you compare the lengths of the bars, it is easy to see that Julie studied for the greatest number of hours on Thursday and the least number of hours on Friday.
A double bar graph is useful for analyzing and comparing two sets of related data. A double bar graph uses vertical or horizontal bars of different heights or lengths to display data. A double bar graph shows 2 sets of related data for a particular value. A key indicates what each bar represents.

A school had a 4-week food drive. Students brought food in cans or boxes. The double bar graph below compares the number of cans to the number of boxes of food students brought each of the 4 weeks in the food drive.

- The scale on the horizontal axis shows the weeks of the food drive.
- The scale on the vertical axis shows the number of cans or boxes of food brought by students.
- The key indicates which bars represent cans and which bars represent boxes.
- By looking at the heights of the bars on the graph, you can see that, except in the second week, students brought more cans than boxes of food each week.
A line graph shows points plotted on a coordinate grid and connected by line segments. The plotted points represent ordered pairs of numbers taken from the data being described by the graph. Line graphs are particularly useful for showing trends, or changes in data over a period of time.

Heavy rainstorms in an area caused local streams and rivers to rise. This line graph shows the depth of a river over the course of 5 days.

![Water Level of a River](image)

The broken line on the vertical axis indicates that the values between 0 and 20 are not included. The relative height of the points can be used to compare the water level of the river each day over the course of 5 days.

Here are some conclusions that can be drawn from this graph:

- Between Monday and Tuesday the water level increased by about 1/2 foot.
- The water level increased between every two days, except between Thursday and Friday.
- The water level reached its highest point on Thursday.
There are 68 musicians in an orchestra's string section. The circle graph below shows the number of people who play the different instruments.

The whole, shown by the circle, represents the total number of stringed instruments, 68. The sections represent the number of people who play different instruments. The size of each section of the circle is determined by the fraction of the whole that represents the number of people playing a certain instrument.

You can use the circle graph to compare the number of people who play each instrument.

- The greatest fraction of people in the string section play the violin. This group is the largest section of the circle.
- The same number of people play the cello as play the viola. These sections of the circle are the same size.
- The number of people who play the string bass (10), the cello (12), and the viola (12) is 34. This is half of the 68 musicians. These sections altogether are half of the circle.
A Venn diagram is used to show how many pieces of data have a certain property in common. The data in each section of a Venn diagram show how many values fall within each category.

This Venn diagram shows the number of students in Jim’s class who went running, swimming, both, or neither at least once last week.

- The diagram includes a rectangle and two overlapping circles. The rectangle represents the universal set that is made up of all the students in Jim’s class.
- The 4 shown in the rectangle but not in the circles represents the 4 students who did not go running or swimming last week.
- The part where the two circles overlap shows the number of students who went both running and swimming: 112.
- The two circles show that 38 students went running only and 83 students went swimming only.

You could use the Venn diagram to find that a total of $38 + 112 = 150$ students in all went running last week.
As part of a history project, Kelly asked 100 people who they thought was the best U.S. president before 1965. Here are the results of Kelly’s survey:

<table>
<thead>
<tr>
<th>Best U.S. President</th>
<th>Number of Votes</th>
</tr>
</thead>
<tbody>
<tr>
<td>George Washington</td>
<td>☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐</td>
</tr>
<tr>
<td>Thomas Jefferson</td>
<td>☐ ☐ ☐ ☐ ☐ ☐</td>
</tr>
<tr>
<td>Abraham Lincoln</td>
<td>☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐</td>
</tr>
<tr>
<td>Franklin D. Roosevelt</td>
<td>☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐</td>
</tr>
<tr>
<td>John F. Kennedy</td>
<td>☐ ☐ ☐ ☐ ☐ ☐</td>
</tr>
</tbody>
</table>

Kelly organized the survey results in a circle graph. Does the circle graph represent Kelly’s data correctly?

A total of _________ people were surveyed.

George Washington was chosen by _________ out of 100 people, or _________%.

Thomas Jefferson was chosen by _________ out of 100 people, or _________%.

Abraham Lincoln was chosen by _________ out of 100 people, or _________%.

Franklin D. Roosevelt was chosen by _________ out of 100 people, or _________%.
How Can You Draw Conclusions and Make Convincing Arguments About Data?

Sometimes you need to draw conclusions or make convincing arguments about data. To do this, you need to review the data carefully and find evidence that supports your conclusion or argument.

This double bar graph shows the number of boys and girls enrolled in each of several foreign-language classes at a school.

In order to make a convincing argument, you need to have data to support your conclusions.
Here are some conclusions that can be drawn from the data shown in the graph above:

- The number of girls taking German is the same as the number of boys taking German.
  This statement is supported by the fact that the bar representing the number of girls taking German and the bar representing the number of boys taking German are the same height. They are both at 45.

- More girls than boys take Spanish.
  This statement is supported by the fact that the bar representing the number of girls taking Spanish is higher than the bar representing the number of boys taking Spanish. There are 57 girls and 53 boys taking Spanish.

- More boys than girls take French, Latin, Chinese, and Japanese.
  This statement is supported by the fact that the bars for these languages are all higher for boys than for girls.

Incorrect conclusions can also be drawn from the graph. For instance, since there is a total of 244 girls and 265 boys represented on the graph, someone might conclude that boys enjoy studying foreign languages more than girls do. This is not a valid conclusion; there could simply be more boys than girls attending that particular school.

**Try It**

This bar graph shows the favorite vacation places of 260 people surveyed.
Read each statement. Mark a “✓” by the statements that are supported by the data in the graph. Mark an “X” by the statements that are not supported by the data in the graph. State the reason for your choice in the blank below each statement.

<table>
<thead>
<tr>
<th>Statement</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>The number of people who like to vacation in the mountains and the number of people who like to vacation in a national or state park are about the same.</td>
<td>The first statement is supported by the data because the bar lengths are about equal. There are about 27 people who like to vacation in the mountains and about 25 people who like to vacation at a national or state park.</td>
</tr>
<tr>
<td>The number of people who like to vacation at the beach is almost half the number who like to vacation in the mountains.</td>
<td>The bar for people who like to vacation at the beach is actually more than twice as long as the bar for people who like to vacation in the mountains.</td>
</tr>
<tr>
<td>About 25% of the people surveyed like to vacation in a small town.</td>
<td>A total of 260 people were surveyed. The number 65 is 25% of 260. The bar for people who like to vacation in a small town shows about 47 people. This is less than 25% of 260.</td>
</tr>
<tr>
<td>Slightly fewer than (\frac{1}{5}) of the people surveyed like to vacation at the lake.</td>
<td>One fifth of the 260 people surveyed is 52 people. The bar for people who like to vacation at the lake shows 50 people. The value 50 is slightly less than 52.</td>
</tr>
</tbody>
</table>
**How Can You Describe a Set of Data?**

One way to describe a set of data is to find the difference between the greatest and least numbers in the set. This difference is called the range of the data.

Look at this set of data. What is its range?

74, 105, 91, 82, 74, 52, 98

Find the difference between the greatest and least values in the set.

105 − 52 = 53

The range of this set of data is 53.

A set of data can also be described by the median, mode, and mean, each of which tells about the central tendency of the data.

The median of a set of data is the middle value of all the numbers. To find the middle value, list the numbers in order from least to greatest or from greatest to least. If there are two middle values, their average is the median.

Look again at the set of data above. What is its median?

- The numbers in order are as follows:
  105, 98, 91, 82, 74, 74, 52
- Cross out numbers starting at either end to help you find the middle value.
  105, 98, 91, 82, 74, 74, 52

The median of this set of data is 82.

Look at this set of data. What is its median?

52, 163, 45, 49, 62, 270

- The numbers in order are as follows:
  45, 49, 52, 62, 163, 270
- Cross out numbers starting at either end. The middle values are 52 and 62.
  45, 49, 52, 62, 163, 270

The average of the middle values is $(52 + 62) ÷ 2$, or $114 ÷ 2$, or 57.

The median of this set of data is 57.
The **mode** of a set of data is the value or values that occur most often in the set. If all the values in a set of data appear the same number of times, the set has no mode.

Look at this set of data. What is its mode?

$55, 60, 60, 75, 75, 75, 80$

The value 75 occurs three times. The value 60 occurs only twice, and each other number occurs only once.

The mode of this set of data is 75.

Look at this set of data. What is its mode?

$455, 450, 655, 455, 345, 655, 789, 453, 455, 655, 875$

The values 455 and 655 each occur three times. Each of the other values appears only once.

This set has two modes, 455 and 655.

The **mean** of a set of data is the average of the values. To find the mean, add all the values and then divide the sum by the number of values in the set.

Look at this set of data. What is its mean?

$140.5, 111.6, 85.8, 59.2, 105.4$

The mean of these 5 values is their sum divided by 5.

$140.5$

$111.6$

$85.8$

$59.2$

$+ 105.4$

$= 502.5$

$502.5 \div 5 = 100.5$

The mean of this set of data is 100.5.
The list below shows the number of months each of several employees worked for a company.

12, 17, 21, 2, 19, 16

Is the mean of this set of data greater than the median?

- Find the mean, or the average, of this set of data.
  \[
  \frac{(21 + 19 + 17 + 16 + 12 + 2)}{6} = \frac{87}{6}
  \]
  \[87 \div 6 = 14.5\]
  The mean is 14.5 months.

- Find the median of this set of data.
  \[
  2, 12, 16, 17, 19, 21
  \]
  \[(16 + 17) \div 2 = 16.5\]
  The median is 16.5 months.

No, the mean of this set of data is not greater than the median. It is less.

**Try It**

Patti works in the school library. She made this bar graph to show the numbers of books checked out during one school week.

What are the range, median, mean, and mode of this set of data?

- Find the range of this set of data.
  The greatest number of books checked out was _________.
  The least number of books checked out was _________.
  The range of this set of data is _________.
How Do You Know Which Measure of Data Is Appropriate to Use in a Given Situation?

Each measure of data (range, mean, median, and mode) gives a different detail about the data. Often one of these measures is more appropriate to use than another. When choosing a measure of data to use, look at the question you are trying to answer. Then think about what each measure would tell you about the data. Finally, choose the measure that best answers the question being asked.
The time it takes Gill to drive to work varies from day to day. He is trying to get to work on time more often. The table below shows his driving times in minutes for one workweek. Which measure of data best helps him analyze how much his driving times vary?

<table>
<thead>
<tr>
<th>Day</th>
<th>Time (minutes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monday</td>
<td>25</td>
</tr>
<tr>
<td>Tuesday</td>
<td>28</td>
</tr>
<tr>
<td>Wednesday</td>
<td>22</td>
</tr>
<tr>
<td>Thursday</td>
<td>24</td>
</tr>
<tr>
<td>Friday</td>
<td>29</td>
</tr>
</tbody>
</table>

Different measures of data tell you different things.

- The range of this set of data is $29 - 22 = 7$.
  
  This information tells how widely Gill's driving times vary. The difference between the shortest and longest times it took him to drive to work is 7 minutes. This information helps Gill because it tells him that he needs to allow for the 7 minutes’ variation in his travel time and leave early enough to be sure to get to work on time.

- This set of data has no mode because each value occurs only once. This measure is not particularly useful.

- The mean of this set of data is the average of the times it takes him to drive to work.
  
  $$\frac{25 + 28 + 22 + 24 + 29}{5} = 25.6$$

  Gill's average driving time is 25.6 minutes. This information does not tell how widely his times vary. This might be a good measure of the data if Gill is trying to determine his average daily driving time.

- The median of this set of data is 25.

  This information does not tell how widely his times vary. This measure may be useful if Gill wants to know about how much time per day he spends driving to work.

The range of this set of data best helps Gill analyze how much his driving times vary.
**Try It**

Ron's older brother does push-ups every morning. He did the following numbers of push-ups each day for the past 10 days.

105, 110, 115, 115, 110, 115, 105, 110, 115, 100

Which measure of data tells the most frequently occurring number of push-ups he did per day?

The mean of the data tells ________________________________
________________________________________________________.

The median of the data tells ________________________________
________________________________________________________.

The mode of the data tells ________________________________
________________________________________________________.

The range of the data tells ________________________________
________________________________________________________.

The ____________ is the measure of data that tells the most frequently occurring number of push-ups Ron's brother did per day.

The mean of the data tells the average of the values. The median of the data tells the middle value. The mode of the data tells the value that occurs most often. The range of the data tells the difference between the least number of push-ups and the greatest number of push-ups. The mode is the measure of data that tells the most frequently occurring number of push-ups Ron's brother did per day.

**Now practice what you’ve learned.**
Question 42

For his lunch Lee can choose from three types of sandwiches: chicken, ham, or peanut butter. He can choose from two drinks: milk or juice. Which list represents the sample space of sandwiches and drinks Lee can choose for lunch?

A  Chicken and milk; chicken and juice; chicken and ham; ham and milk; ham and juice; peanut butter and milk
B  Chicken and milk; ham and milk; peanut butter and milk
C  Chicken and ham; peanut butter and milk; ham and juice
D  Chicken and milk; chicken and juice; ham and milk; ham and juice; peanut butter and milk; peanut butter and juice

Question 43

Hayden has 3 coins. Which of the following diagrams shows all the possible outcomes if Hayden tosses all 3 coins at the same time?
Question 44

A survey was done to count the number of fans attending different sporting events. The results of the survey are shown in the table below.

<table>
<thead>
<tr>
<th>Event</th>
<th>Number in Attendance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soccer match</td>
<td>50</td>
</tr>
<tr>
<td>Track meet</td>
<td>7</td>
</tr>
<tr>
<td>Swim meet</td>
<td>25</td>
</tr>
<tr>
<td>Baseball game</td>
<td>10</td>
</tr>
<tr>
<td>Football game</td>
<td>8</td>
</tr>
</tbody>
</table>

Which type of graph would best represent the data shown in the table?

A  Venn diagram  
B  Line graph  
C  Circle graph  
D  Bar graph

Answer Key: page 151

Question 45

Which set of data has a mean of 10 and a range of 8?

A  4, 10, 8, 12, 12  
B  6, 14, 10, 9, 13  
C  7, 11, 12, 12, 8  
D  9, 9, 7, 10, 15

Answer Key: page 152

Question 46

John has scores of 88, 73, 90, 85, and 93 on five math quizzes. What score must John earn on the next math quiz to have a mean quiz score of exactly 88?

A  88  
B  95  
C  99  
D  100

Answer Key: page 152
Question 47
A survey asked a group of 1,000 people what type of bread they like best. The table below shows their preferences.

<table>
<thead>
<tr>
<th>Type of Bread</th>
<th>Number of People</th>
</tr>
</thead>
<tbody>
<tr>
<td>White</td>
<td>684</td>
</tr>
<tr>
<td>Wheat</td>
<td>212</td>
</tr>
<tr>
<td>Rye</td>
<td>83</td>
</tr>
<tr>
<td>Potato</td>
<td>21</td>
</tr>
</tbody>
</table>

Which graph best represents the data in this table?
Question 48

The graph below shows the outside temperature at various times of day.

Which of the following statements is NOT supported by the line graph?

A  The range of temperatures shown is about 21 degrees.
B  The temperature increased most between 10 A.M. and 11 A.M.
C  The temperature at 11 A.M. was 70°F.
D  The highest temperature shown is 73°F.
Question 49
The table shows how many grams of a given substance can be dissolved in 100 milliliters of water at different temperatures.

<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>Amount of Substance Dissolved (grams)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>30</td>
<td>15</td>
</tr>
<tr>
<td>75</td>
<td>37</td>
</tr>
<tr>
<td>100</td>
<td>51</td>
</tr>
<tr>
<td>150</td>
<td>75</td>
</tr>
</tbody>
</table>

Based on the table, which conclusion seems most reasonable?

A  As the temperature increases, the amount of substance dissolved decreases.
B  As the temperature decreases, the amount of substance dissolved increases.
C  As the temperature increases, the amount of substance dissolved increases.
D  As the temperature changes, there is no effect on the amount of substance dissolved.

Question 50
Eight students took a quiz. Their scores were 75, 83, 91, 75, 100, 55, 40, and 65. Which measure of data would change if a ninth student took the quiz and scored 68?

A  Range
B  Median
C  Mode
D  Mean

Question 51
Organizers of a conference know the number of people who attended their conference each of the past 7 years: 585, 725, 688, 700, 590, 630, and 700. Which measure of data would NOT help them determine approximately how many people to expect at their conference this year?

A  Range
B  Median
C  Mode
D  Mean
For this objective you should be able to

- apply mathematics to everyday experiences and activities;
- communicate about mathematics; and
- use logical reasoning.

**How Can You Use Mathematics to Solve Everyday Problems?**

Many situations in everyday life involve mathematics. Suppose you want to compute the number of hours you need to work to save enough money to buy something. Or suppose you want to estimate the number of feet of ribbon needed in order to decorate for a party. Solutions to problems like these require the use of mathematics.

Solving problems involves more than just numerical computation; logical reasoning and careful planning also play important roles. Here are some steps to follow when solving problems:

- Understand the problem. Organize the information you are given and identify exactly what you must find. You may need information that is not given in the problem, such as a formula. You may be given information that is not needed in order to solve the problem.

- Make a plan. After you have organized the information, decide how to use this information to find an answer. Think about the math concepts that apply to the situation. Identify the order in which you will find new information and the formulas or equations you will use to find it.

- Carry out the plan. After you have chosen a problem-solving strategy, use the strategy to work toward a solution to the problem. Go step-by-step through your plan, writing down important information at each step.

- Check to see whether your answer is reasonable. Check to see whether your answer makes sense. Does it answer the question asked? Is it stated in the correct units? Is it reasonable? You can estimate the solution and then compare the estimate to your answer. They should be approximately equal.
On Friday, Marissa bought a new tape player for $78. On Saturday, the tape player went on sale for $59. What percent could Marissa have saved if she had waited until Saturday to purchase the tape player?

Understand the problem.
- What information do you already have?
  - The original price, $78
  - The sale price, $59
- What do you want to know?
  - The percent of the original price that was taken off

Make a plan.
- To find the amount taken off, subtract the sale price from the original price.
- To find the percent of the original price that was taken off, divide the amount taken off by the original price.
- Express the quotient as a percent.

Carry out the plan.
- $78 - $59 = $19 taken off
- Express the discount as a fraction of the original price.
  \[
  \frac{\text{Amount taken off}}{\text{Original price}} = \frac{19}{78}
  \]
- \[19 \div 78 = 0.24\]
- \[0.24 = 24\%\]

The sale price was 24% off the original price.

Check your answer to see whether it is reasonable.
One way to check for reasonableness is to use estimation. Compare your estimate to the answer you obtained.
- The original price was $78. This rounds to $80.
- The sale price was $59. This rounds to $60.
- $80 - $60 = $20. About $20 was taken off the original price.
  \[
  \frac{\text{Amount taken off}}{\text{Original price}} = \frac{20}{80} = \frac{1}{4} = 25\%
  \]
- About 25% was taken off of the original price.

The estimate of 25% is close to the answer of 24%. Therefore, 24% is a reasonable answer.
Celia is planning a budget for her back-to-school shopping. She is saving $25 per week. So far she has saved $200. She plans to spend about $500 on back-to-school items. She is planning to purchase items that range in price from $2 to $150. Will Celia have enough money for her back-to-school shopping?

- What do you know?
  Celia saves $25 per week. She has already saved $200. She plans to spend $500 on back-to-school items.

- What do you want to know?
  Will Celia have enough money for her back-to-school shopping?

The problem does not give any information about when Celia will go shopping. The number of weeks she has left to save the rest of the $500 is missing from the problem. Additional information is needed in order to solve this problem, so you cannot answer the question.

Try It

Mark wants to put a radio in a rectangular box with a volume of 540 cubic inches. The rectangular box has a length of 10 inches and a width of 9 inches. Mark's radio is a rectangular prism measuring 9 inches by 8 inches by 9 inches. Will Mark's radio fit into the box?

You know two dimensions of the box, the _________ and _________ .

You need to find the _________ of the box in order to know if the radio will fit into the box.

To find the height of the box, \( h \), write an equation using the ______________ of the box. Substitute the values you know into the equation and solve for the ________________.

\[
V_{\text{box}} = \text{______} \cdot \text{______} \cdot \text{______}
\]

\[
\text{______} = \text{______} \cdot \text{______} \cdot h
\]

\[
\text{______} = \text{______} \cdot \text{______} \cdot \text{______}
\]

The height of the box is ________ inches. Mark's radio ________________ fit into the box because the height of the radio is ________ than the ________ of the box.
You know the length and width of the box. You need to find the height of the box. To find the height, $h$, write an equation using the volume of the box. Substitute the values you know into the equation and solve for the height.

$$V_{\text{box}} = l \cdot w \cdot h$$

$$540 = 10 \cdot 9 \cdot h$$

$$540 = 90h$$

$$6 = h$$

The height of the box is 6 inches. Mark’s radio will not fit into the box because the height of the radio is greater than the height of the box.

**What Is a Problem-Solving Strategy?**

A problem-solving strategy is a plan for solving a problem. Different strategies work better for different types of problems. Sometimes you can use more than one strategy to solve a problem. As you practice solving problems, you will discover which strategies you prefer and which work best in various situations.

Some problem-solving strategies include

- drawing a picture;
- making a table;
- looking for a pattern;
- working a simpler problem; and
- guessing and checking;
- working backwards;
- acting it out;

One problem-solving strategy is to work backwards. Begin with what you are told in the problem and then work backwards to find the solution.

Jenny is 5 times as old as Will. Will is 4 years younger than Louis. Louis is half as old as John. John is 12 years old. How old is Jenny? Start with the information that you know and work backwards to find Jenny’s age.

John is 12 years old.

Louis is half as old as John. Since $12 \cdot \frac{1}{2} = 6$, Louis is 6 years old.

Will is 4 years younger than Louis. Since $6 - 4 = 2$, Will is 2 years old.

Jenny is 5 times as old as Will. Since $5 \cdot 2 = 10$, Jenny is 10 years old.
Each Saturday Luis walks the same route to the gym. At 9:15 A.M. he passes the school, which is 2 blocks from his house. At 9:21 A.M. he reaches the library. At this point Luis is 4 blocks from the gym, or halfway from his house to the gym. If Luis walks at the same rate all the way to the gym, what time will he arrive?

- Find the distance from the school to the library.
  At 9:21 A.M. Luis has 4 more blocks to go, and he is halfway to the gym. The distance from his house to the library must also be 4 blocks. Therefore, the distance from the school to the library must be 2 blocks.

- Find the rate at which Luis walks.
  At 9:15 A.M. Luis passes the school, and at 9:21 A.M. he reaches the library.
  \[
  9:21 \text{ A.M.} - 9:15 \text{ A.M.} = 6 \text{ minutes}
  \]
  It takes Luis 6 minutes to walk 2 blocks, or 3 minutes to walk 1 block.

- Use this rate to find the time he will arrive at the gym.
  Luis is 4 blocks from the gym. If it takes him 3 minutes to walk 1 block, it will take him 12 minutes to walk 4 blocks. Twelve minutes after 9:21 A.M. is 9:33 A.M.

Luis will arrive at the gym at 9:33 A.M.
Sometimes you need to combine strategies to solve a problem. Often when you make a table from the data in a problem, you can then look for a pattern to solve the problem.

You know that $2^2 = 4$, $2^3 = 8$, $2^4 = 16$, and so on. If you expand the pattern further, what digit is in the ones place of $2^{13}$ when $2^{13}$ is written as a whole number?

One way to solve this problem is to calculate $2^{13}$, but that is a very large number! Another way is to calculate some of the powers of 2 and see whether there is a pattern for the digits in the ones place.

You can continue the pattern found in the table of 2, 4, 8, 6.

<table>
<thead>
<tr>
<th>Power</th>
<th>Process</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2^1$</td>
<td>$2$</td>
<td>2</td>
</tr>
<tr>
<td>$2^2$</td>
<td>$2 \cdot 2$</td>
<td>4</td>
</tr>
<tr>
<td>$2^3$</td>
<td>$2 \cdot 2 \cdot 2$</td>
<td>8</td>
</tr>
<tr>
<td>$2^4$</td>
<td>$2 \cdot 2 \cdot 2 \cdot 2$</td>
<td>16</td>
</tr>
<tr>
<td>$2^5$</td>
<td>$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$</td>
<td>32</td>
</tr>
<tr>
<td>$2^6$</td>
<td>$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$</td>
<td>64</td>
</tr>
<tr>
<td>$2^7$</td>
<td>$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$</td>
<td>128</td>
</tr>
<tr>
<td>$2^8$</td>
<td>$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$</td>
<td>256</td>
</tr>
</tbody>
</table>

- Look for a pattern in the digits in the ones place of the powers of 2.
- Notice the pattern of the digits in the ones place.
  
  2, 4, 8, 6, 2, 4, 8, 6

- It is not necessary to calculate the exact values of $2^9$, $2^{10}$, $2^{11}$, $2^{12}$, and $2^{13}$ in order to find the digit in the ones place of $2^{13}$.

  You can continue the pattern found in the table of 2, 4, 8, 6.

  $2^9$ would have a 2 in the ones place.
  $2^{10}$ would have a 4 in the ones place.
  $2^{11}$ would have an 8 in the ones place.
  $2^{12}$ would have a 6 in the ones place.
  $2^{13}$ would have a 2 in the ones place.

The digit in the ones place of $2^{13}$ is 2.
How Do You Change Words into Math Language and Symbols?

It is important to be able to rewrite a problem using mathematical language and symbols. The words used in the problem will give you clues about what operations to use.
Rani is selling peanut and popcorn products for a school fundraiser. Each peanut product costs $5, and each popcorn product costs $7. There is an additional 5% tax and $2 shipping fee on each order. Write an expression to find the total cost of an order that includes 3 peanut products and 4 popcorn products.

Each peanut product costs $5. The total for 3 peanut products is represented by $3 \cdot 5$.

Each popcorn product costs $7. The total for 4 popcorn products is represented by $4 \cdot 7$.

Add the two totals together to find the total cost of the peanut and popcorn products.

$$3 \cdot 5 + 4 \cdot 7$$

Multiply the sum by 5%, or 0.05, to find how much tax to add.

$$0.05(3 \cdot 5 + 4 \cdot 7)$$

Add the cost of the products, the tax, and the $2 shipping fee together.

$$(3 \cdot 5 + 4 \cdot 7) + 0.05(3 \cdot 5 + 4 \cdot 7) + 2$$

This expression can be used to find the total cost of an order containing 3 peanut products and 4 popcorn products.

Sometimes real-life situations can best be described using mathematical language and symbols. First represent the quantities involved and then use an equation or formula that accurately reflects the relationships represented in the problem.

A circular circus tent measures 500 feet around its circumference. The tent has an aisle directly through the middle. Write an equation to find the length of the aisle.

The length of the aisle is the diameter of the circle formed by the tent.

The Mathematics Chart lists the formula for the circumference of a circle as follows: $C = \pi d$.

Use the formula to write an equation that can be used to find the length of the diameter. Substitute 500 for $C$ and solve for $d$.

$$500 = \pi d$$

$$\frac{500}{\pi} = \pi d$$

$$\frac{500}{\pi} = d$$

The equation $d = \frac{500}{\pi}$ can be used to find the length of the aisle.
Try It

During a school recycling drive, Jeff’s science class collected 40 pounds of aluminum cans. Ashley’s science class collected twice as many pounds of cans as Jeff’s class collected. Ben’s science class collected 5 more pounds of cans than Ashley’s class collected. Write an expression that can be used to find the total number of pounds of aluminum cans collected by all three classes.

Jeff’s class collected _______ pounds of cans.

Ashley’s class collected _______ as many pounds of cans as Jeff’s class. The expression _______ • _______ represents the number of pounds of cans collected by Ashley’s class.

Ben’s class collected _______ more pounds of cans than Ashley’s class. The expression _______ • _______ + _______ represents the number of pounds of cans collected by Ben’s class.

Combine the expressions for each class to create an expression that gives the total number of pounds of aluminum cans collected by all three classes.

The expression _______ + (_______ • _______ ) + (_______ • _______ + _______ ) can be used to find the total number of pounds of aluminum cans collected by all three classes.

---

Jeff’s class collected 40 pounds of cans. Ashley’s class collected twice as many pounds of cans as Jeff’s class. The expression 2 • 40 represents the number of pounds of cans collected by Ashley’s class. Ben’s class collected 5 more pounds of cans than Ashley’s class. The expression 2 • 40 + 5 represents the number of pounds of cans collected by Ben’s class. The expression 40 + (2 • 40) + (2 • 40 + 5) can be used to find the total number of pounds of cans collected.
How Can You Use Logical Reasoning as a Problem-Solving Tool?

Logical reasoning is thinking of something in a way that makes sense. Thinking about mathematics problems involves logical reasoning.

You can use logical reasoning to find patterns in a set of data. You can then use those patterns to draw conclusions about the data that can be used in order to solve problems.

Finding patterns involves identifying characteristics that numbers or objects have in common. Look for the pattern in different ways.

A sequence of geometric objects may have some property in common. For example, they may all be quadrilaterals or all have right angles.

Mark is training for a one-mile race. During the first month of his training program, his average time was 9 minutes 18 seconds. During the second month his average time was 7 minutes 18 seconds. During the third month his average time was 6 minutes 18 seconds. During the fourth month his average time was 5 minutes 48 seconds. If the pattern continues, what will his average time be during his sixth month of training?

Between the first and second months, Mark improved his time by 2 minutes. Between the second and third months, Mark improved his time by 1 minute. Between the third and fourth months, Mark improved his time by 30 seconds.

The pattern started with a 2-minute improvement in the second month. Half of 2 minutes is 1 minute. The third month showed a 1-minute improvement. Half of 1 minute is 30 seconds. The fourth month showed a 30-second improvement.

If the pattern continues, the fifth month should show a 15-second improvement to an average time of 5 minutes 33 seconds. The sixth month should show a 7.5-second improvement to an average time of 5 minutes 25.5 seconds.
Nathan’s TV Store keeps track of its sales each week. The table below summarizes the store’s sales for the past 4 weeks.

<table>
<thead>
<tr>
<th>Week</th>
<th>Sales (nearest $)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$3,250</td>
</tr>
<tr>
<td>2</td>
<td>$3,088</td>
</tr>
<tr>
<td>3</td>
<td>$2,934</td>
</tr>
<tr>
<td>4</td>
<td>$2,787</td>
</tr>
</tbody>
</table>

If this pattern continues, about how many dollars in sales can Nathan’s TV Store expect in week 6?

- Look for a pattern in the table. The sales are decreasing. Are they decreasing by a constant number of dollars?

  No, the sales are not decreasing by a constant amount.

- Are sales decreasing by a constant rate?

  \[
  \frac{162}{3,250} \approx 0.05 \quad \frac{154}{3,088} \approx 0.05 \quad \frac{147}{2,934} \approx 0.05
  \]

  Yes, each week sales are decreasing at a rate of about 0.05, or 5%.

- Use this rate of decrease to predict the sales for week 5.

  \[
  0.05 \cdot 2,787 = 139.35
  \]

  The sales should decrease by about $139 in week 5. Week 5 sales should be about $2,787 – $139 = $2,648.

- Use the predicted sales for week 5 to predict the sales for week 6.

  \[
  0.05 \cdot 2,648 = 132.40
  \]

  The sales should decrease by about $132 in week 6. Week 6 sales should be about $2,648 – $132 = $2,516.

If this pattern continues, Nathan’s TV Store can expect about $2,516 in sales in week 6.
Question 52
Which of these statements is best supported by the data in the table?

<table>
<thead>
<tr>
<th>Name</th>
<th>Salary</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anna</td>
<td>$12,000</td>
</tr>
<tr>
<td>Scott</td>
<td>$26,000</td>
</tr>
<tr>
<td>Elise</td>
<td>$27,000</td>
</tr>
<tr>
<td>Tomás</td>
<td>$12,000</td>
</tr>
<tr>
<td>Jerome</td>
<td>$75,000</td>
</tr>
<tr>
<td>Tasha</td>
<td>$106,000</td>
</tr>
</tbody>
</table>

A  Jerome’s salary is three times Scott’s salary.
B  Tasha’s salary is more than the other five salaries combined.
C  If Elise’s salary increases by 5%, the mean of the salaries will change to $43,225.
D  The mode of the salaries is 50% of the median of the salaries.

Question 53
Which of these statements is best supported by the data in the table?

<table>
<thead>
<tr>
<th>State</th>
<th>Number of Boys</th>
<th>Number of Girls</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arkansas</td>
<td>16</td>
<td>18</td>
</tr>
<tr>
<td>Texas</td>
<td>29</td>
<td>33</td>
</tr>
<tr>
<td>Florida</td>
<td>11</td>
<td>7</td>
</tr>
<tr>
<td>Alabama</td>
<td>27</td>
<td>15</td>
</tr>
<tr>
<td>Louisiana</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>New Mexico</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>Oklahoma</td>
<td>11</td>
<td>10</td>
</tr>
</tbody>
</table>

A  About 20 percent of the students who attended the conference were from Louisiana.
B  The ratio of boys to girls attending the conference was 21:23.
C  For each state the number of girls who attended the conference was higher than the number of boys who attended.
D  The greatest percent of students at the conference were from Texas.
Question 54

A circle has a diameter of 30 centimeters. Which could be the first step in finding the area of the circle?

A. Find the circumference of the circle  
B. Find the square of the diameter  
C. Multiply the diameter by 2  
D. Divide the diameter by 2

Question 55

Mario purchases 5 equally priced videotapes and a $20 DVD. He spends a total of $45. Which of the following equations can be used to determine \( v \), the cost in dollars of each videotape?

A. \( 5v - 20 = 45 \)  
B. \( 45 \div 5v = 20 \)  
C. \( 5v + 20 = 45 \)  
D. \( 5v \cdot 20 = 45 \)

Question 56

Tonya needs to be at a party at 7:00 P.M. Before leaving, she wants to exercise for \( \frac{3}{4} \) hour and clean her room for 25 minutes. It will take her 50 minutes to get ready for the party and 15 minutes to walk there. Which of the following would be the best strategy to use to find the latest time Tonya should start doing these tasks in order to be at the party at 7:00 P.M.?

A. Work backwards using the information given  
B. Measure the distance to the party in miles  
C. Calculate the rate at which Tonya walks  
D. Multiply the rate Tonya walks by the distance
Question 57

Mr. Jackson is planting three 25-by-25-foot flower beds with petunias and begonias. He spaces the flowers so that each petunia is in a 6-by-6-inch square and each begonia is in a 1-by-1-foot square. The petunias need fertilizer spread at a rate of 50 pounds per 500 square feet. The begonias need fertilizer spread at a rate of 50 pounds per 1,000 square feet. Fertilizer is sold in 50-pound bags. What additional information does Mr. Jackson need in order to determine how much fertilizer to purchase?

A  The price of 1 bag of fertilizer
B  The number of petunias and begonias he will plant
C  The total number of square yards in all three flower beds
D  No additional information is necessary.

Question 58

One of the angles of a parallelogram measures 63°. What are the measures of the other three angles of the parallelogram?

A  63°, 63°, and 63°, because all the angles of a parallelogram are always congruent
B  63°, 27°, and 27°, because consecutive angles of a parallelogram are complementary and the sum of the measures of the angles of a parallelogram is 180°
C  63°, 117°, and 117°, because consecutive angles of a parallelogram are supplementary and the sum of the measures of the angles of a parallelogram is 360°
D  Cannot be determined
Objective 1

Question 1 (page 39)

A Incorrect. The fraction \( \frac{5}{8} \) is equivalent to \( \frac{10}{16} \). This is smaller than \( \frac{11}{16} \), so the wrench would be too small for the bolt.

B Incorrect. The fraction \( \frac{7}{16} \) is smaller than \( \frac{11}{16} \), so the wrench would be too small for the bolt.

C Correct. Rewrite \( \frac{3}{4} \) using 16 as the denominator: \( \frac{3}{4} \cdot \frac{4}{4} = \frac{12}{16} \). Compare \( \frac{11}{16} \) to \( \frac{12}{16} \).

Since \( 11 < 12 \), then \( \frac{11}{16} < \frac{12}{16} \).

Since the \( \frac{11}{16} \)-inch wrench was too small, a \( \frac{3}{4} \)-inch wrench (which is equivalent to \( \frac{12}{16} \)) might be large enough.

D Incorrect. The fraction \( \frac{1}{2} \) is equivalent to \( \frac{8}{16} \). This is smaller than \( \frac{11}{16} \), so the wrench would be too small for the bolt.

Question 2 (page 39)

A Correct. The part of the class that has traveled outside their home state is \( \frac{15}{25} \). Choice A \( \frac{10}{15} \) is not equivalent to this value, so it does not represent the part of the class that has traveled outside their home state.

Question 3 (page 39)

D Correct. The model shows 8 dots by 8 dots. There are a total of 64 dots: \( 8 \cdot 8 = 8^2 = 64 \). The square root of 64 is 8, because \( 8 \cdot 8 = 64 \). Therefore, the model can be used to find \( \sqrt{64} \).

Question 4 (page 39)

B Correct. You need to separate \( 4 \frac{1}{3} \) pounds into equal \( \frac{3}{8} \)-pound servings. Use division to separate a whole into equal parts. The expression \( 4 \frac{1}{3} \div \frac{3}{8} \) shows the number of \( \frac{3}{8} \)-pound hamburger patties that can be made from \( 4 \frac{1}{3} \) pounds of ground beef.

Question 5 (page 40)

A Correct. Tammy owns \( \frac{1}{3} \) of the business. Subtract to find the remaining part of the business.

\[ 1 - \frac{1}{3} = \frac{2}{3} \]

María and her three brothers equally share the remaining \( \frac{2}{3} \) of the business.

Use division to separate \( \frac{2}{3} \) into 4 equal parts.

\[ \frac{2}{3} \div 4 \]

Multiply by \( \frac{1}{4} \), the reciprocal of \( \frac{4}{1} \). Remember that 4 is the same as \( \frac{4}{1} \).

\[ \frac{2}{3} \cdot \frac{1}{4} = \frac{2 \cdot 1}{3 \cdot 4} = \frac{2}{12} = \frac{1}{6} \]

María owns \( \frac{1}{6} \) of the business.

Question 6 (page 40)

B Correct. One way to find the change in temperature is to first see what the total change in temperature was and then decide whether it was a positive change (an increase in temperature) or a negative change (a decrease in temperature).

The temperature went from \( -5°F \) to \( 41°F \). That change can be modeled on a number line.

\[ 5 + 41 = 46 \]

The temperature increased by \( 46°F \). The integer \( +46 \) represents this change.

Question 7 (page 40)

D Correct. To find the cost of 1 pound of cheese, divide the cost of 2 pounds of cheese by 2.
The expression $\frac{7.50}{2}$ represents the cost of 1 pound of cheese, which is the unit cost.

To find the cost of 4.5 pounds of cheese, multiply 4.5 by the unit cost.

The expression $\frac{7.50}{2} \cdot 4.5$ represents the cost of 4.5 pounds of cheese.

**Question 8 (page 40)**

C Correct. The order of operations requires that any operations in parentheses be performed first. There are no parentheses in this expression, so the next step is to simplify any exponents. Since $3^3$ is in exponential form, it should be performed first.

**Question 9 (page 41)**

D Correct. You need to know the unit rate for both walkers.

Peter walked 4 miles in 50 minutes. Divide 4 miles by 50 minutes to find how many miles he walked per minute.

$$\frac{4}{50} = 0.08$$

Peter’s walking rate was 0.08 mile per minute.

Jan walked 3 miles in 30 minutes. Divide 3 miles by 30 minutes to find how many miles she walked per minute.

$$\frac{3}{30} = 0.1$$

Jan's walking rate was 0.1 mile per minute. Subtract to find the difference in their walking rates.

$$0.1 - 0.08 = 0.02$$

Peter’s walking rate was 0.02 mile per minute slower than Jan’s.

**Question 10 (page 41)**

B Correct. To find how many boxes of cereal the Thompson family eats in 1 week, multiply the amount eaten per day by 7 days.

$$\frac{3}{8} \cdot 7 = \frac{3 \cdot 7}{8} = \frac{21}{8}$$

The fraction $\frac{21}{8}$ is equal to $2\frac{5}{8}$ or 2.625.

Since the Thompson family cannot buy 2.625 boxes of cereal, they would need to buy 3 boxes to have enough cereal for 1 week.

**Question 11 (page 41)**

D Correct. If you convert all fractions to decimals or write them with the lowest common

denominator, only answer choice D lists them from least to greatest.

**Decimal Conversion Method**

$$\frac{3}{16} = 0.1875, \quad \frac{5}{8} = 0.625, \quad \frac{3}{4} = 0.75,$$

$$\frac{13}{16} = 0.8125, \quad \frac{7}{8} = 0.875$$

**Lowest Common Denominator Method**

The lowest common denominator for 4, 8, and 16 is 16.

$$\frac{3}{16}, \quad \frac{5}{8}, \quad \frac{3}{4}, \quad \frac{13}{16}, \quad \frac{7}{8} = \frac{14}{16}$$

**Question 12 (page 42)**

A Correct. In the grid modeling $\frac{3}{4}$, three of the four rows are shaded. And in the grid modeling $\frac{1}{5}$, one of the five columns is shaded. Therefore, the grid modeling the answer should have three rows and one column shaded. In this grid, only three of the 20 squares are shaded in both directions. It models the fraction $\frac{3}{20}$. This set of grids models the number sentence $\frac{3}{4} \cdot \frac{1}{5} = \frac{3}{20}$.

**Objective 2**

**Question 13 (page 63)**

D Correct. Use either the proportion method or the decimal method to find the final cost of the meal.

**Proportion Method**

Remember that percent means per 100.

Set up a proportion using $\frac{20}{100}$ for 20%.

$$\frac{20}{100} = \frac{x}{48.30}$$

Solve the proportion by using cross products.

$$\frac{20}{100} = \frac{x}{48.30}$$

$$966 = 100x$$
\[
\frac{966}{100} = \frac{100x}{100} \\
9.66 = x
\]

The discount on the meal was $9.66.
To find the final cost of the meal, subtract the discount from the cost of the meal.
$48.30 − 9.66 = $38.64

**Decimal Method**
To find the discount, find 20% of the cost of the meal.
First express the percent as a decimal.
20% = 0.20
Then multiply the cost of the meal by 0.20.
$48.30 \cdot 0.20 = 9.66$
The discount on the meal was $9.66.
To find the final cost of the meal, subtract the discount from the cost of the meal.
$48.30 − 9.66 = 38.64$
The final cost of the meal was $38.64 after the coupon was used.

**Question 14 (page 63)**
B Correct. Of the students surveyed, 35 out of 125 have their own e-mail account. To express \( \frac{35}{125} \) as a decimal, divide 35 by 125.
\[
35 \div 125 = 0.28 \\
0.28 = 28\%
\]
Of the students surveyed, 28% have their own e-mail account.

**Question 15 (page 63)**
B Correct. The problem uses two different units of measure. Convert all measurements to pints. The Mathematics Chart shows the number of quarts in a gallon and the number of pints in a quart.
- 1 gallon = 4 quarts; 1 quart = 2 pints
- 1 gallon = (4 quarts) \cdot (2 pints per quart)
- 1 gallon = 8 pints
- 2 gallons = 16 pints
Write two ratios that compare price to volume. Let \( x \) equal the cost of 2 gallons of soy sauce.
The ratio for the 2-gallons (16 pints) of soy sauce is \( \frac{x}{16 \text{ pints}} \).
Write a proportion by setting the two ratios equal to each other. Solve the proportion using cross products.
\[
\frac{1.79}{2} = \frac{x}{16} \\
2x = 28.64 \\
x = 14.32
\]
The cost of 2 gallons of soy sauce is $14.32.

**Question 16 (page 63)**
A Correct. The triangle in the scale model and the triangle formed by the ramp are similar figures. The ratios of their corresponding sides are equal.
The height of the ramp (4 feet) corresponds to 3 inches in the model.
The length of the ramp (\( x \) feet) corresponds to 8 inches in the model.
Write a proportion by setting the ratios of the corresponding sides equal to each other and then solve the proportion.
\[
\frac{4}{3} = \frac{x}{8} \\
3x = 32 \quad \text{Use cross products.} \\
x = \frac{32}{3} \quad \text{Divide both sides of the equation by 3.} \\
x = 10\frac{2}{3} \quad \text{Divide 32 by 3:} \ 32 \div 3 = 10 \text{ R2.} \\
The number 10 is the whole-number part; 2 is the fractional part of the mixed number.

The ramp will be 10\( \frac{2}{3} \) feet long.

**Question 17 (page 64)**
D Correct. In similar figures, the ratios of corresponding sides are equal.
\( \frac{XY}{PQ} \) corresponds to \( \frac{YZ}{QR} \).
\[
\frac{XY}{PQ} = \frac{YZ}{QR}
\]
The proportion \( \frac{5}{12.5} = \frac{9}{x} \) can be used to find the length of \( QR \).
**Mathematics Answer Key**

**Question 18 (page 64)**

**D Correct.** According to the formula chart on page 9 of this guide, the circumference of a circle is \( C = 2\pi r \), where \( r \) is the radius of the circle. We can solve this equation for \( r \) by dividing both sides by \( 2\pi \):

\[
\frac{C}{2\pi} = \frac{2\pi r}{2\pi} \implies r = \frac{C}{2\pi}.
\]

For this reason \( r = \frac{18}{\pi} \), which names the same value as \( \frac{18}{\pi} \).

**Question 19 (page 65)**

**C Correct.** The Mathematics Chart shows that 1 quart = 2 pints.

The \( x \)-coordinates and corresponding \( y \)-coordinates of the ordered pairs in the correct graph should show the same relationship.

The graph shows points at (1, 2), (2, 4), (3, 6), (4, 8), (5, 10), and (6, 12). The \( x \)-coordinates and corresponding \( y \)-coordinates are in the ratio 1:2.

Only this graph has \( x \)-coordinates and corresponding \( y \)-coordinates in this ratio.

**Question 20 (page 66)**

**A Correct.** The \( x \)-axis of the graph is the number of seed packets. The \( y \)-axis of the graph is the number of seeds. The graph shows points at (2, 30), (3, 45), (5, 75), (6, 90), and (8, 120). The point (2, 30), for example, shows that 2 packets contain 30 seeds. This table shows the number of packets and the number of seeds in this same relationship.

<table>
<thead>
<tr>
<th>Position</th>
<th>( 3(n + 1) )</th>
<th>Value of Term</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3(1 + 1)</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>3(2 + 1)</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>3(3 + 1)</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>3(4 + 1)</td>
<td>15</td>
</tr>
<tr>
<td>5</td>
<td>3(5 + 1)</td>
<td>18</td>
</tr>
</tbody>
</table>

The rule works for each term in the sequence 6, 9, 12, 15, 18, . . .

**Question 21 (page 67)**

**C Correct.** To see if a rule applies to a sequence of numbers, apply the rule to each number’s position in the sequence.

Sequence: 6, 9, 12, 15, 18, . . .

<table>
<thead>
<tr>
<th>Position</th>
<th>( 3(n + 1) )</th>
<th>Value of Term</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3(1 + 1)</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>3(2 + 1)</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>3(3 + 1)</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>3(4 + 1)</td>
<td>15</td>
</tr>
<tr>
<td>5</td>
<td>3(5 + 1)</td>
<td>18</td>
</tr>
</tbody>
</table>

The rule works for each term in the sequence 6, 9, 12, 15, 18, . . .

**Question 22 (page 67)**

**C Correct.** Remove like quantities from each side.

\[
3x = 12
\]

Divide both sides of the equation by 3.

\[
\triangle = \nabla + \nabla + \nabla
\]

\[
x = 4
\]

Therefore, \( x = 4 \) is the solution to the equation.

**Question 23 (page 67)**

**D Correct.** To find the average score for 5 games, add the 5 scores and divide by 5. Let the unknown fifth score be represented by \( x \).

Represent the average score, 148, as the sum of the five scores divided by 5.

\[
\frac{128 + 145 + 139 + 157 + x}{5} = 148
\]

**Question 24 (page 94)**

The correct answer is 74.5. Two angles are complementary if the sum of their measures is 90°.

To find the complement of a 15.5° angle, subtract 15.5° from 90°.

\[
90° - 15.5° = 74.5°
\]

The complement of \( \angle Y \) has a measure of 74.5°.
Question 25 (page 94)

C Correct. Corresponding parts are in the same relative position.
Since \( MN \) is the hypotenuse, the side opposite the right angle of triangle \( MNR \), the corresponding side of triangle \( MBA \) must be \( MB \), the side opposite the right angle of triangle \( MBA \). \( MB \) is the side opposite the right angle, so \( MB \) corresponds to \( MN \).

Question 26 (page 94)

A Incorrect. A rectangle is a special parallelogram in which all angles are congruent. This is not true of all parallelograms.
B Incorrect. A rhombus is a special parallelogram in which all sides are congruent. This is not true of all parallelograms.
C Incorrect. A rectangle is a special parallelogram in which adjacent sides are perpendicular to each other. This is not true of all parallelograms.
D Correct. Parallelograms are quadrilaterals in which opposite sides are parallel.

\[
\text{In parallelogram } LMNO, \text{ side } LM \text{ is parallel to side } ON, \text{ and side } MN \text{ is parallel to side } LO.
\]
A parallelogram always has opposite sides that are parallel.

Question 27 (page 94)

B Correct. A pyramid has only one base. The base of an octagonal pyramid is an octagon.
A pyramid also has triangular faces. Since an octagon has 8 sides, the pyramid must have 8 triangular faces.

Question 28 (page 95)

The correct answer is 110. Consecutive angles in a parallelogram are supplementary. The sum of their measures is 180°.

To find the supplement of a 70° angle, subtract 70° from 180°.

\[
180° - 70° = 110°
\]

\[
m\angle F = 110°
\]

<table>
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<tr>
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<th>( \times )</th>
<th>( \div )</th>
<th>( % )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>10</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>0.8</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td>0.9</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>9</td>
<td>1.0</td>
<td></td>
</tr>
</tbody>
</table>

Question 29 (page 95)

C Correct. Corresponding sides of similar triangles are proportional. Corresponding parts are in the same relative position. Since \( MN \) is the hypotenuse, the side opposite the right angle of triangle \( MNR \), the corresponding side of triangle \( MBA \) must be \( MB \), the side opposite the right angle. \( MA \) corresponds to \( MR \), and \( BA \) to \( NR \).

Only the proportion \( \frac{6}{3} = \frac{10}{x} \) compares corresponding sides.

Question 30 (page 96)

B Correct. The coordinates of point \( R \) are \((-2, 3)\).
The \( x \)-coordinate of point \( R \) is \(-2\). The \( x \)-coordinate of the new point is the same.
The \( x \)-coordinate of the new point is \(-2\).
The \( y \)-coordinate of point \( R \) is \(3\). The \( y \)-coordinate of the new point is 6 units down from \(3\). The number \(-3\) is 6 units down from \(+3\).
The \( y \)-coordinate of the new point is \(-3\).
The coordinates of the new point are \((-2, -3)\).

Question 31 (page 96)

D Correct. The coordinates of point \( F \) are \((7, 8)\).
Moving 1 unit to the right makes the new \(x\)-coordinate \(7 + 1 = 8\).
Moving down 7 units makes the new \(y\)-coordinate \(8 - 7 = 1\).
The new coordinates of point \( F \) will be \((8, 1)\).
Question 32 (page 97)
B Correct. The top view is a square. The front and side views are triangles. This choice is the only one with 1 square as the base and 4 triangles as faces.

Question 33 (page 97)
A Correct. The net shows 6 rectangular faces and 2 hexagonal bases. Prisms have 2 bases; pyramids have only 1 base. The figure must be a prism. The bases are hexagons. The three-dimensional figure is a hexagonal prism.

Question 34 (page 106)
C Correct. The area of the rectangular window is equal to its length times its width.

\[ A = lw = 4.5 \cdot 3 = 13.5 \text{ ft}^2 \]

The area of the circular window is equal to \( \pi \) times the square of its radius. Use 3.14 as an estimate of the value of \( \pi \). The radius of the circle is half the diameter, or 2 feet.

\[ A = \pi r^2 = \pi \cdot 2^2 = 3.14 \cdot 4 = 12.56 \text{ ft}^2 \]

The difference between the area of the rectangle and the area of the circle is

\[ 13.5 - 12.56 = 0.94 \text{ ft}^2. \]

The circular window will have an area of about 0.94 square feet less than the rectangular window.

Question 35 (page 106)
B Correct. The formula for the perimeter of a rectangle is

\[ P = 2l + 2w \]

The width, \( w \), is 7 feet. Let \( l \) equal the length of the rectangle in feet.

\[ P = 2l + 2w \]

\[ 50 = 2l + 2(7) \]

Substitute 50 for \( P \) and 7 for \( w \).

\[ 50 = 2l + 14 \]

Multiply: 2(7) = 14.

\[ 36 = 2l \]

Subtract 14 from both sides of the equation.

\[ 18 = l \]

Divide both sides of the equation by 2.

The formula for the area of a rectangle is \( A = lw \).

The length, \( l \), is 18 feet. The width, \( w \), is 7 feet.

\[ A = lw \]

\[ A = 18 \cdot 7 \]

\[ A = 126 \text{ ft}^2 \]

The area of the deck is 126 square feet.

Question 36 (page 106)
B Correct. Square yards is a unit of area. The dimensions of the room are given in feet. Convert the dimensions to yards. Since there are 3 feet in 1 yard, divide by 3 to find the number of yards.

\[ 22 \div 3 = 7\frac{1}{3} \]

\[ 18 \div 3 = 6 \]

Next find the area using the formula for the area of a rectangle.

\[ A = lw \]

\[ A = 6 \cdot 7\frac{1}{3} \]

\[ A = 44 \text{ yd}^2 \]

Mr. Hall will need to buy 44 square yards of carpet.

Question 37 (page 106)
C Correct. The formula for the volume of a cube is \( V = s^3 \). Each of the boxes has a volume of \( V = s^3 = 4^3 = 4 \cdot 4 \cdot 4 = 64 \). It takes 81 boxes to fill each crate, so the volume of a crate is

\[ 81 \cdot 64 = 5,184 \text{ in}^3 \]

The volume of each crate is 5,184 cubic inches.

Question 38 (page 106)
C Correct. The formula for the circumference of a circle is \( C = 2\pi r \).

Use 3.14 as an approximation for \( \pi \). Let \( r \) equal the radius of the circle.

\[ C = 2\pi r \]

\[ 18 = 2 \cdot 3.14 \cdot r \]

\[ 18 = 6.28r \]

\[ 3 = r \]

The formula for the area of a circle is \( A = \pi r^2 \).

\[ A = 3(3)^2 = 27 \]

The area of the table is approximately 27 square feet.

Question 39 (page 107)
D Correct. To find the total number of ounces of ground beef needed, multiply \( 5\frac{1}{4} \) by 32.

\[ 5\frac{1}{4} \cdot 32 = 168 \text{ ounces} \]

To find the number of pounds in 168 ounces, divide 168 by 16.

\[ 168 \div 16 = 10.5 \text{ pounds} \]
In order to make 32 hamburger patties, Mr. Borders will need to buy 11 pounds of ground beef.

**Question 40 (page 107)**

A Correct. Multiply $2\frac{1}{3}$ by 4 to find the number of cups of grain Victor will feed the 4 horses.

$$2\frac{1}{3} \times 4 = \frac{7}{3} \times 4$$

$$= \frac{7 \times 4}{3}$$

$$= \frac{28}{3}$$

$$= \frac{9\frac{1}{3}}{28 \div 3 = 9 \text{ R1}}$$

Victor will feed the horses $9\frac{1}{3}$ cups of grain. The bag originally contained 128 cups of grain. Subtract to find the difference: $128 - 9\frac{1}{3}$.

$$127\frac{3}{3} - 9\frac{1}{3}$$

Rewrite 128 as $127\frac{3}{3}$ so that you can subtract $\frac{1}{3}$ from it.

$$- \frac{9\frac{1}{3}}{118\frac{2}{3}}$$

After Victor feeds the horses, there will be $118\frac{2}{3}$ cups of grain left in the bag.

**Question 41 (page 107)**

A Correct. The formula for the volume of a rectangular prism is $V = lwh$.

$$V = lwh$$

$$V = 7 \times 2 \times 6\frac{1}{2}$$

$$V = 14 \times 6\frac{1}{2}$$

$$V = 91 \text{ in.}^3$$

The volume of the rectangular prism is 91 cubic inches. The cylinder should have the same volume as the rectangular prism. The formula for the volume of a cylinder is $V = \pi r^2h$. Substitute 3.14 for $\pi$ and 91 for $V$.

$$V = \pi r^2h$$

$$91 \approx 3.14 \times 4^2 \times h$$

$$91 \approx 3.14 \times 16 \times h$$

$$91 \approx 50.24 \times h$$

$$1.8 \approx h$$

The height of the cylinder would be approximately 1.8 inches.

**Objective 5**

**Question 42 (page 126)**

A Incorrect. This choice lists chicken and ham. Lee can choose only one sandwich.

B Incorrect. This choice lists only lunches that have milk as the drink choice. Juice is also a drink choice.

C Incorrect. This choice lists chicken and ham. Lee can choose only one sandwich.

D Correct. Lee has three sandwich choices. For each of the three sandwich choices, Lee has two drink choices. Use a tree diagram to model all the possible combinations.

These are the 6 possible combinations for Lee’s lunch.

**Question 43 (page 126)**

C Correct. The first column represents the outcomes for the first coin: heads or tails. The second column represents the outcomes for the second coin. Notice that these outcomes are repeated in the second column because they are matched with the possible outcomes from the first coin: (H, H), (H, T), (T, H), and (T, T). Again, in the third column, we will have two branches representing the possible outcomes for the third coin. These are repeated since they are matched with the possible outcomes of the other two coins.

**Question 44 (page 127)**

D Correct. The bar graph would best represent the data. Because a bar graph uses bars of different lengths to compare data, it would show a comparison between the number of fans attending each event.
Question 45 (page 127)

A Incorrect. The data have a range of 8 and a mean of 9.2.
B Incorrect. The data have a range of 8 and a mean of 10.4.
C Incorrect. The data have a range of 5 and a mean of 10.
D Correct. The greatest value is 15. The least value is 7. The range is 15 \( - H_11002 \) 7 = 8. The mean is \((9 + 9 + 7 + 10 + 15) \div 5 = 50 \div 5 = 10\).

Question 46 (page 127)

C Correct. Add to find the total points John has after taking 5 quizzes.

\[
88 + 73 + 90 + 85 + 93 = 429
\]

To have a mean score of 88 after taking 6 quizzes, John's total points on 6 quizzes should equal \(6 \times 88 = 528\) points.

John needs a total of 528 points. Right now he has 429.

John must earn \(528 - 429 = 99\) points. John needs 99 points on his next quiz in order to have a mean score of 88.

Question 47 (page 128)

A Incorrect. A line graph is used to show trends, or changes in data over a period of time. It is not a good choice to show preferences for a type of bread.

B Incorrect. A Venn diagram is used to show how many pieces of data have a certain property in common. It is not a good choice to show preferences for a type of bread.

C Correct. A circle graph is used to compare a set of data, such as preferences for a type of bread. This circle graph accurately represents the percent of people who like each type of bread.

D Incorrect. A bar graph is used to compare data, such as preferences for a type of bread. However, this bar graph incorrectly shows potato bread chosen by approximately 212 people, not 21 people.

Question 48 (page 129)

B Correct. The temperature increased 10°F between 9 A.M. and 10 A.M. The temperature increased 5°F between 10 A.M. and 11 A.M.

The statement that the temperature increased most between 10 A.M. and 11 A.M. is not supported by the graph.

Question 49 (page 130)

C Correct. Each temperature is about two times the number of grams of substance dissolved at that temperature. As the temperature increases from 20°C to 150°C, the amount of substance that can be dissolved increases from 10 to 75 grams.

Question 50 (page 130)

A Incorrect. The range would still be the difference between the greatest and least scores, which is \(100 - 40 = 60\).

B Incorrect. The median, the middle score, would still be 75.

C Incorrect. The mode, the most frequently occurring score, would still be 75.

D Correct. The original mean was 73. If a grade of 68 were figured in, the mean would be 72.4.

Question 51 (page 130)

A Correct. The range only tells the organizers the difference between the greatest and least numbers of people who attended. The range for the attendance is \(725 - 585 = 140\). The range value, 140, does not help them determine the approximate number of people to expect this year.

B Incorrect. The median is the middle number in the data. It provides a good indication of how many people may be at the next conference.

C Incorrect. The mode is the most frequently occurring number. It provides a good indication of how many people may be at the next conference.

D Incorrect. The mean of a set of data is the average of the values. It provides a good indication of how many people may be at the next conference.

Objective 6

Question 52 (page 142)

A Incorrect. Scott's salary, $26,000, multiplied by three is $78,000.

B Incorrect. Tasha's salary is $106,000. The sum of the other five salaries is $152,000.
C  Correct. Five percent of Elise’s salary is $1,350. Her new salary would be $28,350. The mean would increase to $43,225.

D  Incorrect. The mode of the salaries is $12,000. The median of the salaries is $26,500. The mode, $12,000, is 50% of $24,000.

Question 53 (page 142)

A  Incorrect. A total of 115 boys and 105 girls, or 220 students, attended the conference. From Louisiana 9 boys and 10 girls, or 19 students, attended. The fraction $\frac{19}{220}$ represents the portion of the students from Louisiana. To find the equivalent percent of $\frac{19}{220}$, divide the numerator by the denominator: $\frac{19}{220} \approx 0.086$. Thus, only about 8.6 percent of the students who attended the conference were from Louisiana.

B  Incorrect. In all, 115 boys and 105 girls attended the conference. The ratio of boys to girls was 115:105, which is equivalent to 23:21, not 21:23.

C  Incorrect. Florida, Alabama, and Oklahoma had more boys than girls attending the conference. New Mexico had the same number of girls and boys attending.

D  Correct. From Texas 29 boys and 33 girls, or 62 students, attended. Texas had the greatest number of students attending the conference. If the number of students attending from Texas was the greatest, then the percent attending from Texas would also be the greatest.

Question 54 (page 143)

D  Correct. The formula for the area of a circle is $A = \pi r^2$. To find the area of a circle if the diameter is given, first divide the diameter by 2 to find $r$, the radius. Then square the radius and multiply the product by $\pi$.

Question 55 (page 143)

C  Correct. Let $v$ represent the cost of 1 videotape. If each videotape is the same price, then 5 videotapes cost $5v$. The cost of the DVD is $20. Add to find the total amount Mario spends. The expression $5v + 20$ represents the total amount he spends. The problem states that Mario spends a total of $45. Write an equation setting the expression and total equal to each other. The equation $5v + 20 = 45$ can be used to find $v$, the cost in dollars of each videotape.

Question 56 (page 143)

A  Correct. Tonya knows the number of minutes each of her activities will take. She also knows when she needs to be at the party. If Tonya subtracts the number of minutes she will spend doing each of the tasks from 7:00 P.M., she will find the latest time she should start doing her tasks. She should work backwards using the information given.

Question 57 (page 144)

B  Correct. Mr. Jackson needs to know how much area will be covered by each type of flower. To determine this, he needs to know how many of each type of flower he will plant.

Question 58 (page 144)

C  Correct. A parallelogram is formed by the intersection of two pairs of parallel lines. Consecutive angles in a parallelogram are supplementary, which means that the measures of the two angles add up to 180°. The sum of the measures of the four angles is always 360°.
Grade 7
Mathematics Chart

LENGTH

<table>
<thead>
<tr>
<th>Metric</th>
<th>Customary</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 kilometer = 1000 meters</td>
<td>1 mile = 1760 yards</td>
</tr>
<tr>
<td>1 meter = 100 centimeters</td>
<td>1 mile = 5280 feet</td>
</tr>
<tr>
<td>1 centimeter = 10 millimeters</td>
<td>1 yard = 3 feet</td>
</tr>
<tr>
<td></td>
<td>1 foot = 12 inches</td>
</tr>
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</table>

CAPACITY AND VOLUME

<table>
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<th>Metric</th>
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<tbody>
<tr>
<td>1 liter = 1000 milliliters</td>
<td>1 gallon = 4 quarts</td>
</tr>
<tr>
<td></td>
<td>1 gallon = 128 fluid ounces</td>
</tr>
<tr>
<td></td>
<td>1 quart = 2 pints</td>
</tr>
<tr>
<td></td>
<td>1 pint = 2 cups</td>
</tr>
<tr>
<td></td>
<td>1 cup = 8 fluid ounces</td>
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MASS AND WEIGHT

<table>
<thead>
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<th>Metric</th>
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<tbody>
<tr>
<td>1 kilogram = 1000 grams</td>
<td>1 ton = 2000 pounds</td>
</tr>
<tr>
<td>1 gram = 1000 milligrams</td>
<td>1 pound = 16 ounces</td>
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TIME

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</tr>
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<td>1 day = 24 hours</td>
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<td>1 hour = 60 minutes</td>
<td></td>
</tr>
<tr>
<td>1 minute = 60 seconds</td>
<td></td>
</tr>
</tbody>
</table>

Continued on the next side
### Grade 7 Mathematics Chart

| Perimeter          |  |  |
|--------------------|------------------------|
| square             | $P = 4s$               |
| rectangle          | $P = 2l + 2w$ or $P = 2(l + w)$ |

| Circumference      |  |  |
|--------------------|------------------------|
| circle             | $C = 2\pi r$ or $C = \pi d$ |

| Area               |  |  |
|--------------------|------------------------|
| square             | $A = s^2$               |
| rectangle          | $A = lw$ or $A = bh$    |
| triangle           | $A = \frac{1}{2} bh$ or $A = \frac{bh}{2}$ |
| trapezoid          | $A = \frac{1}{2} (b_1 + b_2)h$ or $A = \frac{(b_1 + b_2)h}{2}$ |
| circle             | $A = \pi r^2$          |

$B$ represents the Area of the Base of a three-dimensional figure.

| Volume             |  |  |
|--------------------|------------------------|
| cube               | $V = s^3$              |
| rectangular prism  | $V = lwh$ or $V = Bh$  |
| triangular prism   | $V = Bh$               |
| cylinder           | $V = \pi r^2h$ or $V = Bh$|

| Pi                 |  |  |
|--------------------|------------------------|
| $\pi$              | $\pi \approx 3.14$ or $\pi \approx \frac{22}{7}$ |